

# REPORT No. 30.

## EXPERIMENTAL RESEARCH ON AIR PROPELLERS, II.

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### INTRODUCTION.

The purpose of the experimental investigation on the performance of air propellers described in the following report was, in brief, the extension of the field covered by the investigation of 1917 and described in Report No. 14 of the Third Annual Report of the National Advisory Committee for Aeronautics, 1917.

The investigation was divided into five parts, as follows:

(I) Tests under conditions of flight on 16 model propellers of different forms, sections, or pitch ratios from those of Report No. 14.

(II) Tests under conditions of flight on one model propeller of variable pitch.

(III) Tests under conditions of flight on three sets of right and left hand model propellers in series.

(IV) Tests on 12 model propellers to determine brake effect or negative thrust at negative slip.

(V) Standing thrust and power tests of 67 model propellers.

The above program was prepared by Dr. W. F. Durand, chairman of the National Advisory Committee for Aeronautics. The entire investigation was carried on during the summer and fall of 1918 in the aerodynamic laboratory at Stanford University, under the direction of Prof. E. P. Lesley, who is author of the report in its present form. Prof. H. C. Moreno, of Stanford University, rendered valuable assistance in the experimental work and reduction of data. Dr. Durand was called to Europe in January, 1918, and was therefore unable to give personal attention to the tests and preparation of the report.

The Stanford University aerodynamic laboratory and its equipment are fully described in Report No. 14, previously mentioned. Except in details which are hereafter described, the method of observation and reduction of data used in this present report was the same as that previously adopted.

### I. TESTS ON 16 MODEL PROPELLERS.

The propellers chosen for this part of the investigation are shown in the accompanying figure 1. They involve six additional forms ( $F_3$ ,  $F_4$ ,  $F_5$ ,  $F_6$ ,  $F_7$ , and  $F_8$ ), one additional type section ( $S_2$ ) and one additional pitch ratio (1.1).

The characteristics of the six additional forms are shown in the accompanying figures 2, 3, and 4. It may be noted that  $F_3$  is similar to  $F_2$  of Report No. 14, the curved center line being of the same shape but being set back in the direction opposite to rotation 0.55 inch. In  $F_4$  the center line is curved in the opposite direction from that of  $F_3$ , making the leading edge instead of the following edge straight. In  $F_5$  the curved center line is set back 1.1 inches, and  $F_6$  is similar except that the center line is curved in the opposite direction.  $F_7$  has a radial leading edge, while  $F_8$  has a radial following edge.

The additional type sections ( $S_2$ ) are shown in figures 5, 6, 7, and 8. This section has one-half the camber on the driving face of  $S_1$  of Report No. 14. Sections ( $S_1$ ) of  $F_3$ ,  $F_4$ ,  $F_5$ , and  $F_6$  are the same as for  $F_2$ ,  $A_1$ ,  $S_1$  (fig. 20 of Report No. 14). Figure 9 shows sections for  $F_7$  and  $F_8$ . The various characteristics of the 16 models are shown in the following Table I.

TABLE I.—*Characteristics of model propellers.*

No.	Diameter.	Nominal pitch.	Nominal pitch ratio.	Dynamic pitch.	Dynamic pitch ratio.	Mean blade width.	Area symbol A.	Shape of blade form symbol.	Blade section symbol.
	<i>Inches.</i>	<i>Inches.</i>		<i>Inches.</i>					
80.....	36	39.6	1.1	50.9	1.414	.15r	A <sub>1</sub>	F <sub>1</sub>	S <sub>1</sub>
81.....	36	39.6	1.1	48.4	1.343	.20r	A <sub>2</sub>	F <sub>1</sub>	S <sub>1</sub>
82.....	36	39.6	1.1	50.7	1.409	.15r	A <sub>1</sub>	F <sub>2</sub>	S <sub>1</sub>
83.....	36	39.6	1.1	47.2	1.312	.20r	A <sub>2</sub>	F <sub>2</sub>	S <sub>1</sub>
84.....	36	25.2	.7	33.8	.938	.15r	A <sub>1</sub>	F <sub>3</sub>	S <sub>1</sub>
85.....	36	25.2	.7	34.5	.958	.15r	A <sub>1</sub>	F <sub>4</sub>	S <sub>1</sub>
86.....	36	25.2	.7	33.1	.918	.15r	A <sub>1</sub>	F <sub>5</sub>	S <sub>1</sub>
87.....	36	25.2	.7	34.6	.959	.15r	A <sub>1</sub>	F <sub>6</sub>	S <sub>1</sub>
88.....	36	25.2	.7	35.4	.993	.135r	A <sub>3</sub>	F <sub>7</sub>	S <sub>1</sub>
89.....	36	25.2	.7	35.2	.976	.135r	A <sub>3</sub>	F <sub>8</sub>	S <sub>1</sub>
90.....	36	18.0	.5	26.5	.737	.15r	A <sub>1</sub>	F <sub>9</sub>	S <sub>2</sub>
91.....	36	18.0	.5	23.9	.664	.20r	A <sub>4</sub>	F <sub>3</sub>	S <sub>2</sub>
92.....	36	25.2	.7	34.8	.966	.15r	A <sub>1</sub>	F <sub>1</sub>	S <sub>2</sub>
93.....	36	25.2	.7	31.3	.870	.20r	A <sub>4</sub>	F <sub>5</sub>	S <sub>2</sub>
94.....	36	32.4	.9	42.8	1.190	.15r	A <sub>1</sub>	F <sub>2</sub>	S <sub>2</sub>
95.....	36	32.4	.9	39.9	1.109	.20r	A <sub>4</sub>	F <sub>8</sub>	S <sub>2</sub>

The dimensions of the propeller sections are given in the following Table II and figure 10:

TABLE II.—*Dimensions of propeller sections.*

[See figure 10.]

Form, area, and section symbols.	Radius of section.	A B.	A E.	A C B D.	E S.	E H.	E M.	H S.	θ
	<i>Inches.</i>	<i>Inches.</i>	<i>Inches.</i>	<i>Inches.</i>	<i>Inches.</i>	<i>Inches.</i>	<i>Inches.</i>	<i>Inches.</i>	<i>°</i>
F <sub>1</sub> A <sub>1</sub> S <sub>1</sub> .....	4	2.45	0.90	0.05	0.70	1.19	.....	.....	78
	7	2.70	.90	.05	.60	1.01	0.033	.....	69
	10	2.70	.90	.05	.50	.84	.033	.....	60
	13	2.70	.90	.05	.40	.66	.033	.....	51
A <sub>2</sub> S <sub>1</sub> .....	16	2.70	.90	.05	.30	.49	.050	.....	42
	4	3.27	1.30	.05	.70	1.19	.....	.....	78
	7	3.60	1.30	.05	.60	1.01	.033	.....	69
	10	3.60	1.30	.05	.50	.84	.033	.....	60
A <sub>1</sub> S <sub>2</sub> .....	13	3.60	1.30	.05	.40	.66	.033	.....	51
	16	3.60	1.30	.05	.30	.49	.050	.....	42
	4	3.00	1.00	.05	.70	1.19	.....	.....	78
	7	3.22	1.07	.05	.60	1.01	.033	.....	69
A <sub>3</sub> S <sub>2</sub> .....	10	3.14	1.05	.05	.50	.84	.033	.....	60
	13	2.68	.89	.05	.40	.66	.033	.....	51
	16	1.90	.63	.05	.30	.49	.050	.....	42
	4	4.00	1.33	.05	.70	1.19	.....	.....	78
A <sub>4</sub> S <sub>2</sub> .....	7	4.30	1.43	.05	.60	1.01	.033	.....	69
	10	4.18	1.39	.05	.50	.84	.033	.....	60
	13	2.53	1.12	.05	.40	.66	.033	.....	51
	16	2.54	.85	.05	.30	.49	.050	.....	42
A <sub>4</sub> S <sub>1</sub> .....	4	2.34	.78	.05	.70	1.19	.....	0.12	78
	7	2.39	.80	.05	.60	1.01	.....	.....	69
	10	2.04	.68	.05	.50	.84	.....	.....	60
	13	3.02	1.01	.05	.40	.66	.....	.....	51
	16	2.05	.63	.05	.30	.30	.....	.....	62

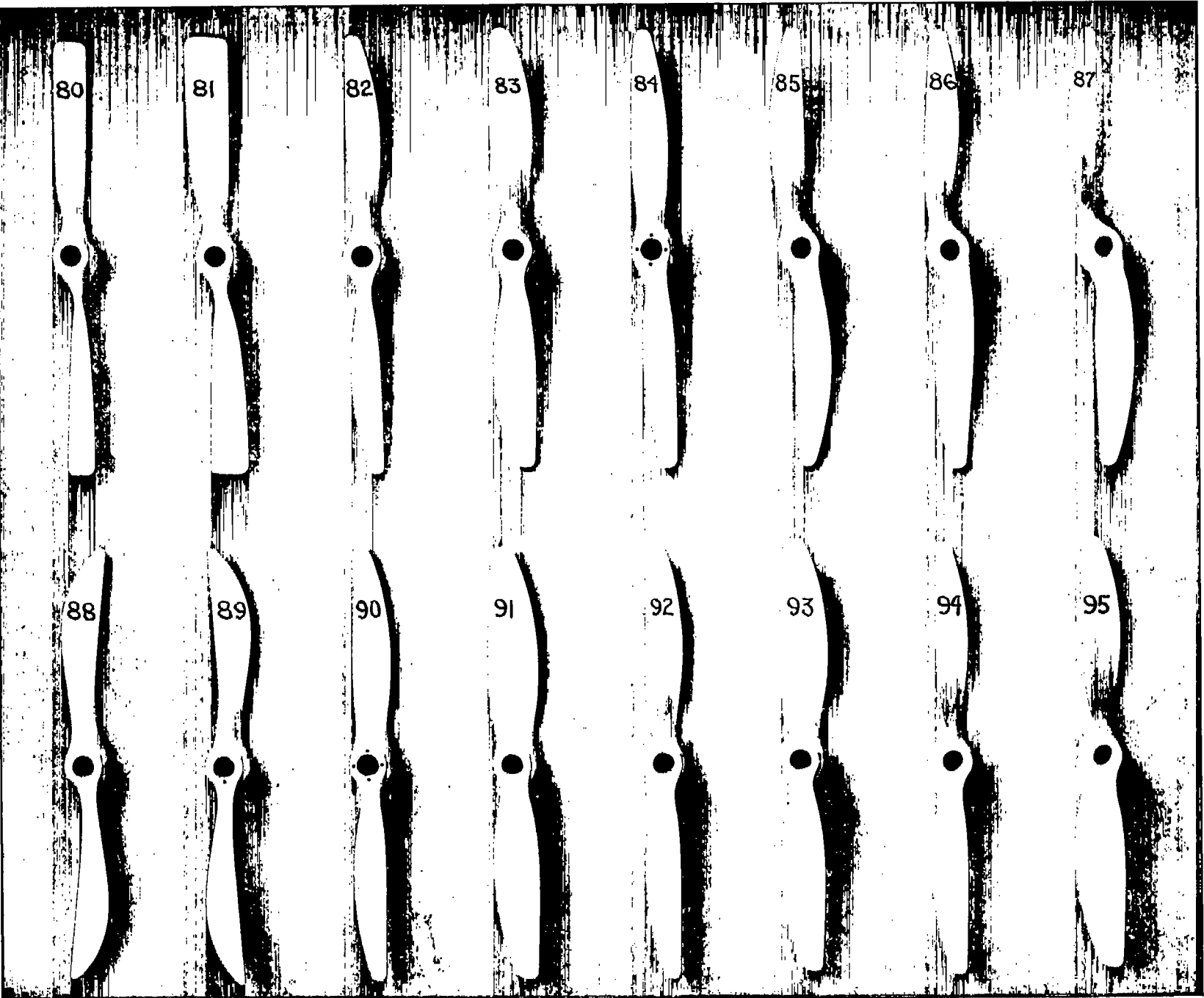
*Construction of models.*—The model propellers were carved from single sticks of Pacific coast sugar pine (*Pinus Lambertiana*) in the manner described in Report No. 14. Further tests and measurements of the models constructed in 1917 show that, if thoroughly seasoned and properly protected by varnish, this wood is well adapted to the purpose.

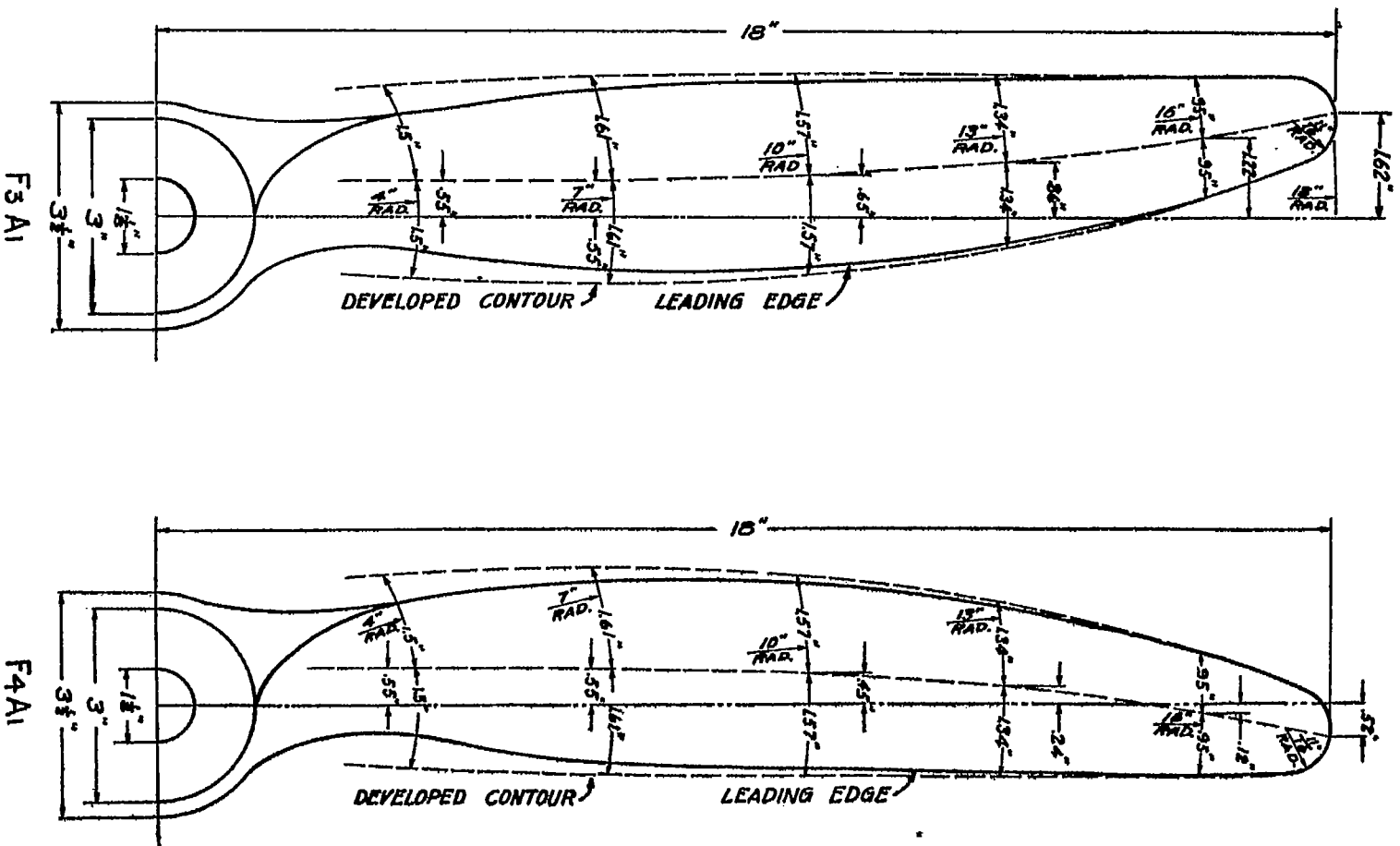
*Method of conducting tests.*—The tests were conducted in the same manner as those of 1917, except in the following particulars:

*Wind velocity.*—In the earlier investigation the wind velocity, except for the action of the model, was constant throughout a test. Two or more velocities were used, the data for very high slip being obtained at the lower velocity.

Although experiment showed that the thrust and torque reduced to coefficients were, at constant slip, independent of velocity; it was thought advisable to use velocities as high as practicable, particularly at low slips, in order that the quantities observed might be large and thus the percentage of error reduced.

Accordingly tests herein described were started with a velocity of about 50 miles (80 km.) per hour. The model propeller was brought to a rotative speed that produced practically no thrust and observations were made. The rotative speed was then increased slightly and another set of observations taken. This was continued until the limit of power for the driving motor was reached. The wind speed was then reduced by small amounts and observations made until the desired range of slip was covered.





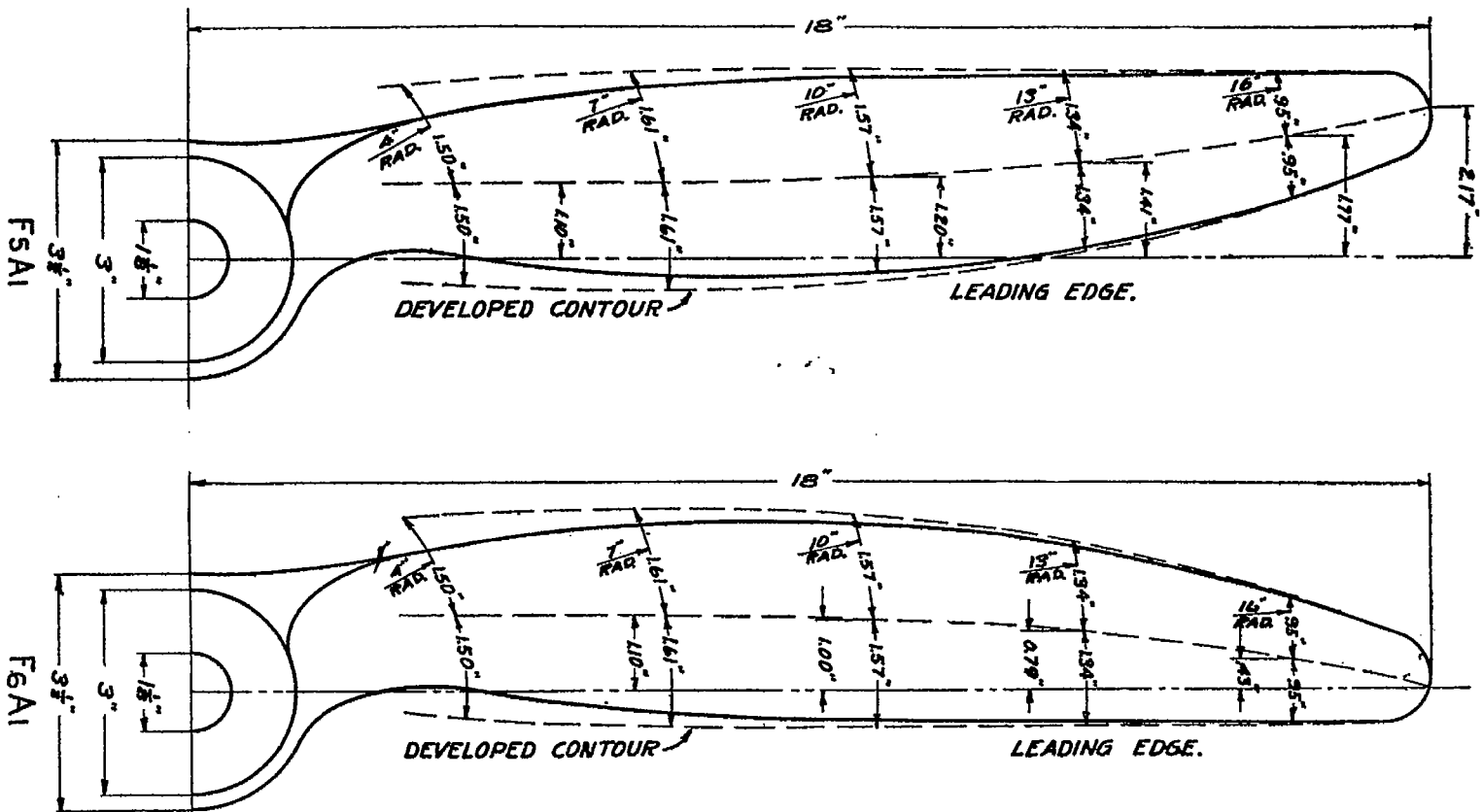


FIG. 8.

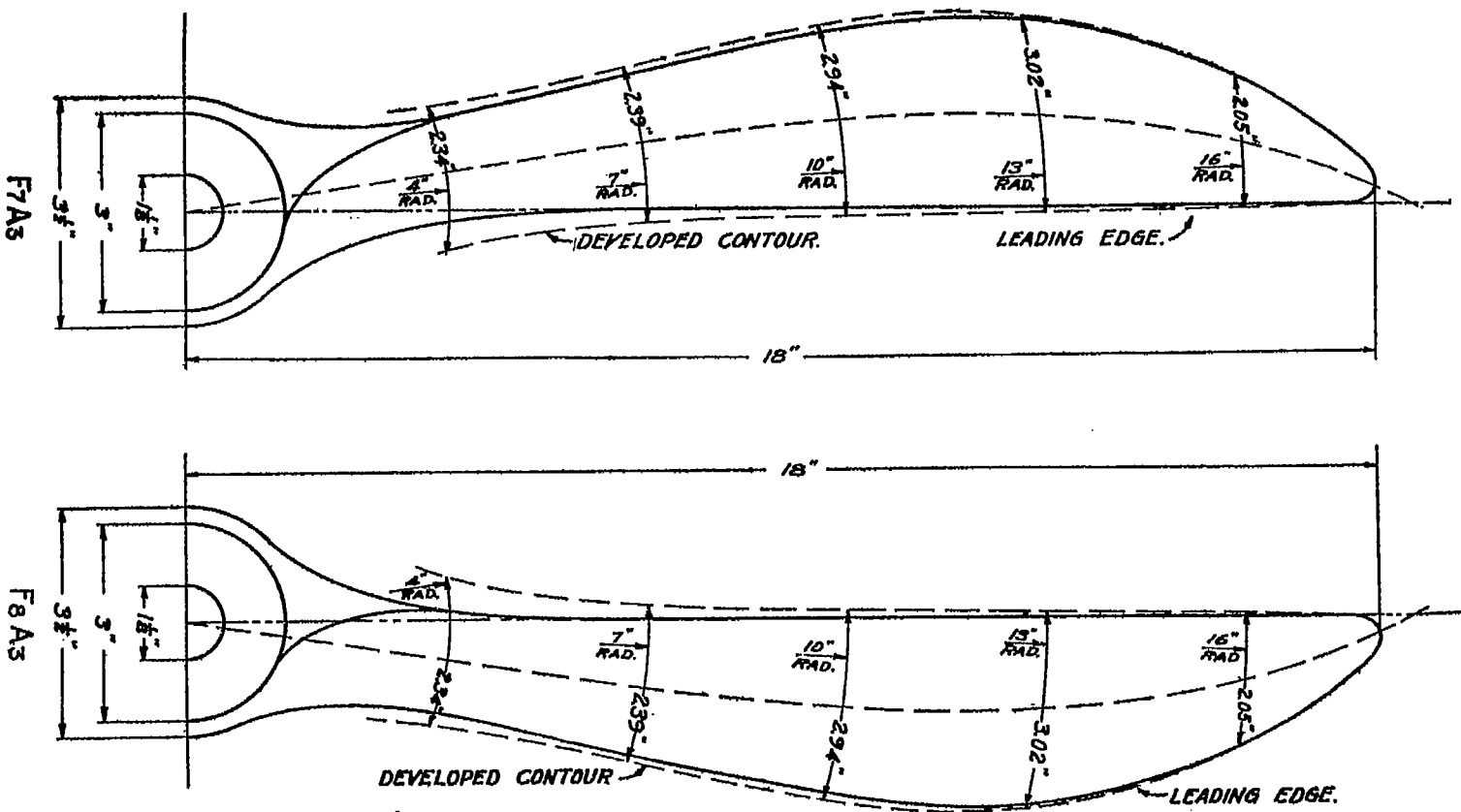


Fig. 4.

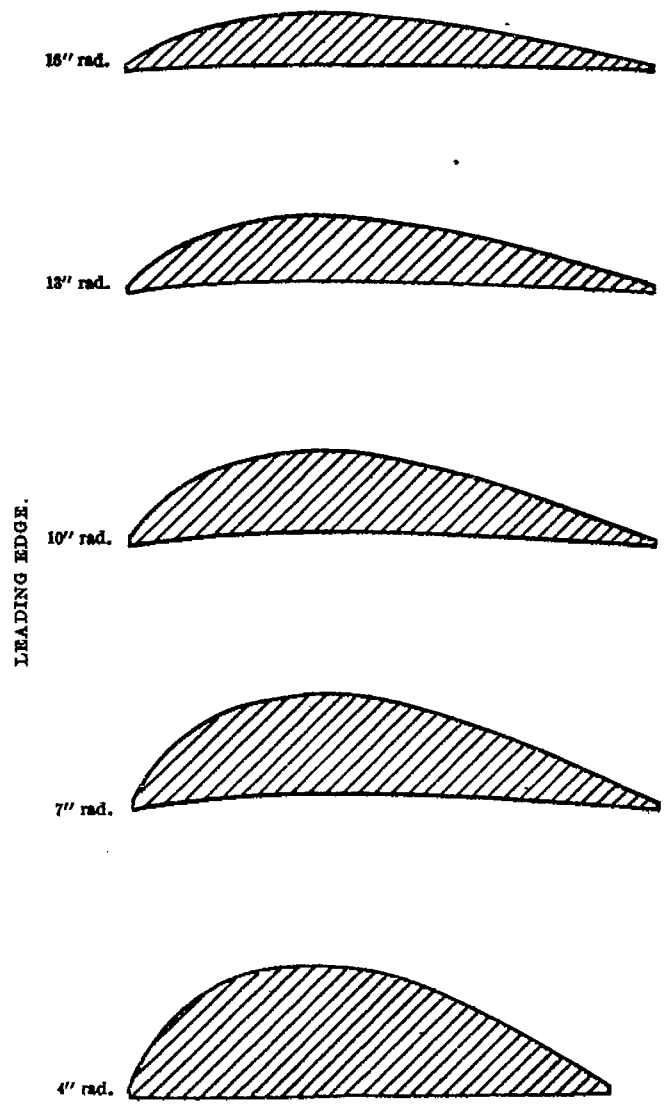


FIG. 5.

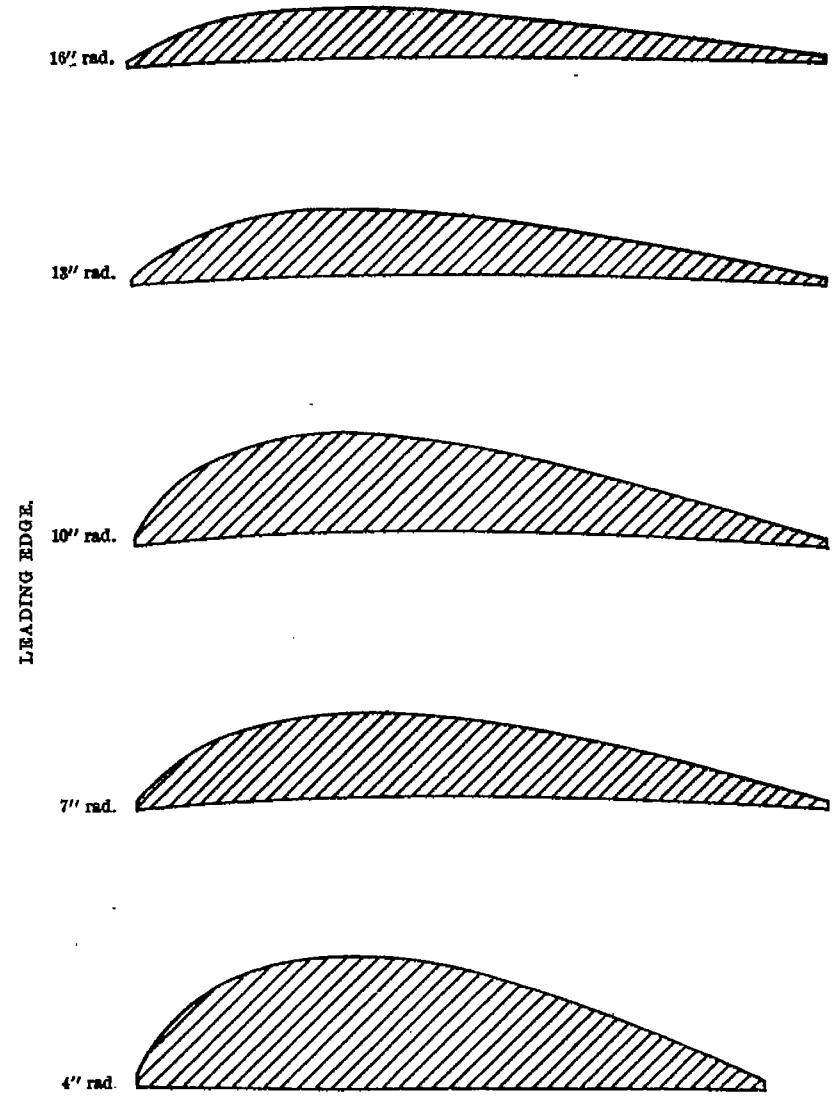


FIG. 6.

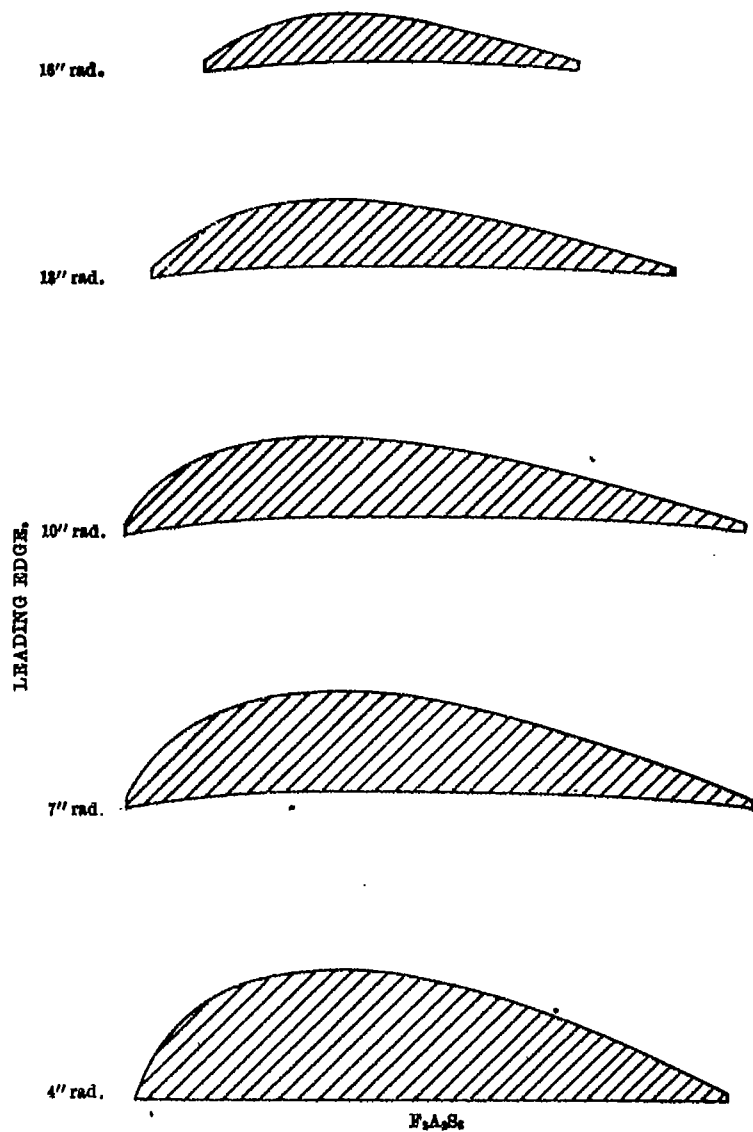


FIG. 7.

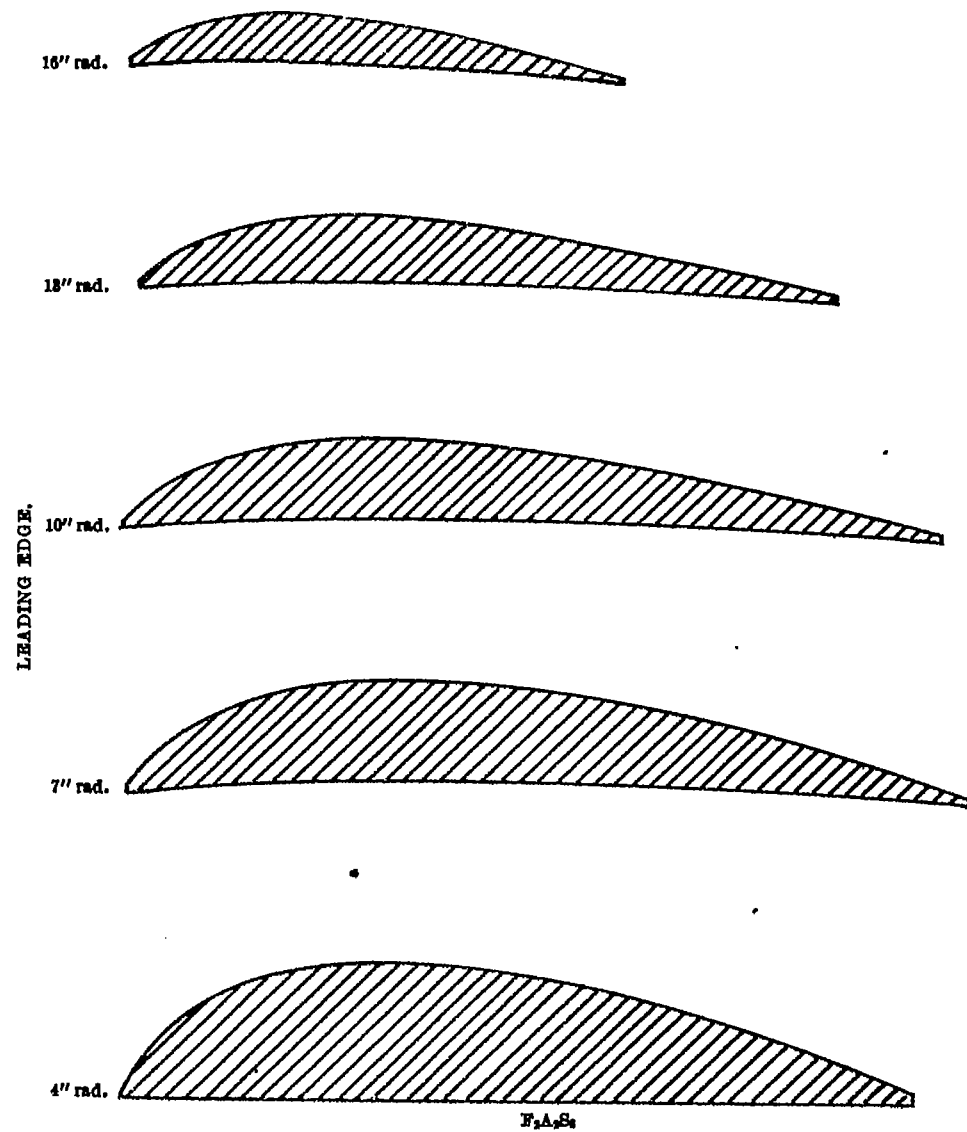
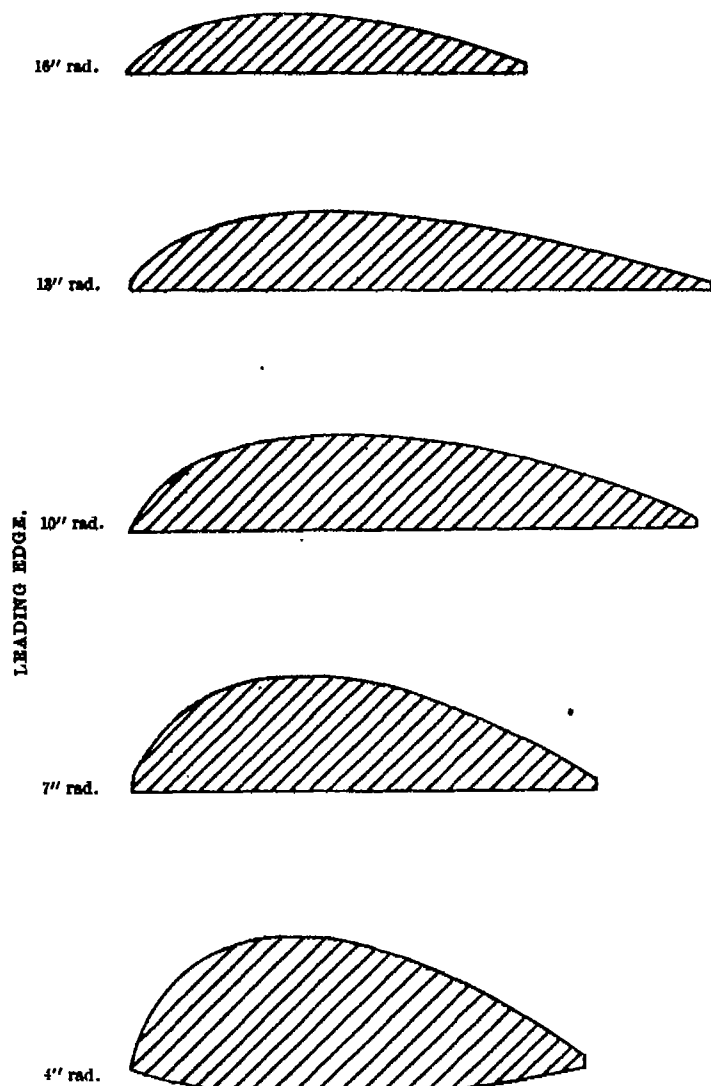
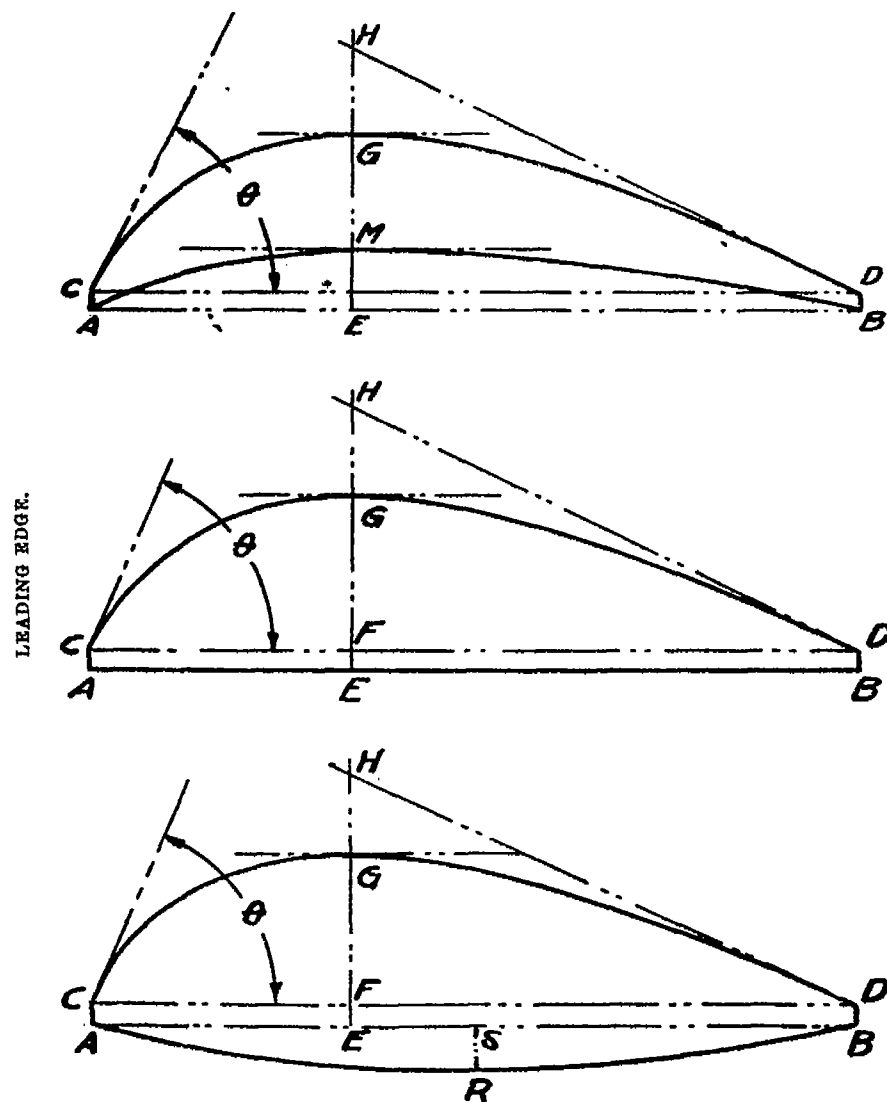


FIG. 8.




$$\mathbf{F}_7 \mathbf{F}_8 \mathbf{A}_3 \mathbf{B}_1$$

**FIG. 9.**



**FIG. 10.**

**Torque.**—In 1917 it was determined that, after running 15 to 20 minutes, the torque moment of the apparatus was practically constant and independent of speed. The practice was therefore to observe the torque zero at the beginning and end of a test in which from 10 to 20 observations occupying about one minute each were made.

Tests at the beginning of this present investigation showed that while the torque zero was practically independent of speed there was a slight variation with time which was not uniform. The following practice was therefore adopted. The driving motor was started and allowed to run until practically a constant torque zero was observed. The model was then put in position and the usual observations made. The model was then removed and an observation of torque zero made at the same number of revolutions.

The discovery of slight variation in the torque zero led to the suspicion that efficiencies determined for the 51 propellers of Report No. 14 might be inaccurate at the lower slips (the vicinity of the maximum efficiency). Consequently all of these propellers were retested by the improved method. It was thus learned that in some cases, particularly for propellers of the higher pitch ratio, the maximum efficiency previously determined was from 1 to 3 per cent in error. The following Table III shows for propellers 1 to 51, Report No. 14, the value of efficiencies for various values of  $\frac{V}{ND}$  as determined by tests during 1918.

TABLE III.—Efficiency of propellers No. 1 to 51.

Propeller No.	$\frac{V}{ND}$														
	.20	.30	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	1.0
1.....	0.341	0.481	0.590	0.630	0.670	0.708	0.730	0.752	0.769	0.780	0.786	0.786	0.780	0.760	0.736
2.....	.530	.486	.583	.620	.660	.695	.726	.745	.763	.776	.781	.779	.783	.717	.650
3.....	.354	.489	.595	.636	.675	.708	.738	.760	.780	.796	.806	.810	.804	.786	.761
4.....	.342	.475	.580	.622	.662	.695	.725	.750	.770	.783	.787	.781	.780	.710	.612
5.....	.372	.517	.620	.661	.697	.726	.745	.753	.745	.715	.685	.655			
6.....	.371	.517	.627	.672	.707	.732	.745	.737	.707	.655	.585	.400			
7.....	.394	.528	.640	.685	.722	.751	.770	.777	.763	.740	.690	.605			
8.....	.375	.522	.635	.680	.715	.744	.760	.759	.738	.683	.580	.210			
9.....	.430	.572	.668	.690	.693	.685	.690	.470							
10.....	.435	.572	.658	.673	.682	.615	.507	.265							
11.....	.440	.591	.680	.701	.707	.690	.644	.560							
12.....	.430	.572	.655	.670	.667	.628	.535	.220							
13.....	.381	.496	.590	.629	.670	.706	.735	.750	.780	.791	.798	.798	.779	.745	.690
14.....	.335	.465	.575	.620	.660	.698	.730	.757	.775	.784	.788	.778	.750	.704	.628
15.....	.368	.495	.598	.642	.683	.717	.747	.770	.787	.798	.808	.800	.790	.760	.710
16.....	.350	.479	.588	.633	.673	.712	.745	.771	.791	.800	.808	.795	.774	.730	.655
17.....	.375	.520	.628	.671	.710	.740	.761	.770	.760	.725	.670	.570			
18.....	.370	.515	.620	.677	.717	.748	.762	.758	.728	.682	.645	.570			
19.....	.390	.540	.650	.695	.730	.759	.776	.781	.770	.740	.670	.550			
20.....	.382	.519	.629	.674	.714	.745	.765	.771	.758	.713	.615	.365			
21.....	.414	.562	.665	.693	.708	.673	.613	.472							
22.....	.414	.555	.642	.659	.641	.582	.464	.210							
23.....	.424	.575	.671	.685	.667	.612	.490								
24.....	.429	.575	.665	.680	.671	.615	.480	.140							
25.....	.302	.433	.548	.596	.636	.671	.700	.728	.744	.752	.755	.750	.738	.706	.665
26.....	.320	.455	.564	.610	.650	.685	.715	.740	.759	.768	.769	.743	.704	.642	.530
27.....	.341	.478	.578	.620	.660	.691	.719	.742	.759	.760	.771	.768	.746	.708	.649
28.....	.334	.462	.564	.608	.647	.683	.714	.740	.752	.775	.782	.774	.741	.680	.570
29.....	.347	.492	.601	.648	.676	.700	.714	.716	.709	.680	.681	.655			
30.....	.365	.495	.601	.645	.680	.706	.721	.720	.691	.630	.480	.190			
31.....	.371	.509	.605	.649	.684	.714	.733	.740	.727	.665	.641	.553			
32.....	.367	.503	.610	.655	.694	.722	.739	.738	.701	.634	.520	.328			
33.....	.398	.535	.630	.673	.700	.725	.743	.748							
34.....	.410	.535	.630	.673	.700	.725	.743	.748							
35.....	.404	.541	.627	.645	.642	.611	.550	.445							
36.....	.400	.530	.614	.629	.622	.582	.498	.320							
37.....	.306	.431	.545	.594	.635	.672	.703	.729	.745	.758	.760	.757	.741	.710	.656
38.....	.312	.445	.560	.610	.651	.690	.720	.745	.760	.760	.747	.717	.660	.595	.480
39.....	.345	.478	.581	.625	.660	.692	.720	.741	.753	.760	.770	.783	.740	.700	.630
40.....	.320	.452	.560	.608	.650	.694	.719	.741	.758	.767	.765	.751	.722	.671	.585
41.....	.345	.480	.590	.635	.674	.703	.722	.735	.709	.661	.601	.480			
42.....	.345	.480	.602	.650	.684	.712	.725	.721	.690	.620	.490	.265			
43.....	.371	.510	.614	.658	.690	.717	.730	.730	.710	.675	.616	.510			
44.....	.359	.500	.605	.645	.680	.701	.708	.697	.665	.572	.427	.180			
45.....	.400	.535	.601	.606	.600	.546	.470	.350							
46.....	.398	.533	.609	.615	.600	.530	.420	.230							
47.....	.415	.545	.609	.614	.601	.565	.490	.384							
48.....	.401	.541	.605	.605	.579	.515	.405	.215							
49.....	.365	.512	.631	.679	.714	.741	.759	.764	.760	.735	.691	.617			
50.....	.380	.530	.642	.685	.720	.740	.752	.752	.735	.695	.630	.525			
51.....	.380	.525	.636	.678	.703	.730	.745	.749	.735	.702	.645	.540			

Where the above efficiency is different from that shown in the curves of Report No. 14, the difference is due mainly to a less net torque caused by a slight rise in zero torque. Quantitatively the error was small, as may be shown by the following illustration:

In Report No. 14 the maximum efficiency of propeller No. 1 is 0.765 at  $\frac{V}{ND} = 0.79$ . With a wind velocity of 60 feet per second (about 40 miles per hour) and with  $Q_c = 0.6$  and  $T_c = 0.365$ , with air density 0.075 pounds per cubic foot, and with a diameter of 3 feet, the torque would be  $\frac{0.6 \times 3600 \times 27 \times 0.075}{1000} = 4.38$  foot-pounds. The efficiency of propeller No. 1 at  $\frac{V}{ND} = 0.79$  has been determined to be 0.785, which with the same thrust, velocity, etc., requires a torque coefficient = 0.585, or a torque of 4.27 foot-pounds, the difference being 0.11 foot-pounds.

A further possible source of error in the efficiency curves of Report No. 14 was the method of calculating efficiency from faired curves of  $T_c$  and  $Q_c$ . The difference in the above example, between  $Q_c = 6$  and  $Q_c = 0.585$  is barely distinguishable on the scale used. In the above check tests efficiency was calculated directly from the data for each observation. It may be noted that the efficiencies (as shown by Table III) for expanding pitch propellers ( $P_2$ ) 13 to 24 and 37 to 48 are in general somewhat greater than for uniform pitch propellers ( $P_1$ ) 1 to 12 and 25 to 36. There are but few exceptions to this rule. Other general conclusions of Report No. 14 as to relative efficiency are unchanged.

#### REDUCTION OF DATA.

The same general method in reduction of data followed in Report No. 14 has been followed in this present investigation. With the exception that metric units are used throughout, the data as presented are in the same form.

$V$  = Velocity in meters per second.

$N$  = Revolutions per second.

$D$  = Diameter in meters.

$\Delta$  = Density of air in kilograms per cubic meter.

$T$  = Thrust in kilograms.

$Q$  = Torque in kilogram-meters.

$P_e$  = Effective power or power absorbed by propellers in kg. meters per second.

$P_u$  = Useful power or work done in overcoming resistance to forward motion of plane in kg. meters per second.

$\rho$  = Efficiency.

It should be noted that  $\frac{V}{ND}$ , the slip coefficient, is independent of the units if they are homogeneous. Therefore, in this present report, the values of  $\frac{V}{ND}$  are the same as for Report No. 14. The coefficients  $T_c = \frac{100 \times T}{\Delta V^2 D^2}$ ,  $Q_c = \frac{1000 \times Q}{\Delta V^2 D^3}$ ,  $\frac{P_e}{\Delta N^2 D^5}$  and  $\frac{P_u}{\Delta N^2 D^5}$  are likewise independent of the units used if they are absolute and homogeneous. Since gravity units are employed these coefficients have in the metric system a value  $g$  in feet divided by  $g$  in meters = (3.28) times that when English units are used.

In Plates I to XII are shown diagrams giving values of the thrust and torque coefficients and of the efficiency for all of the 16 models tested. The use of these diagrams is fully explained in Report No. 14.

The practice in Report No. 14 was to reduce the observations to data in the form of thrust and torque coefficients  $T_c$  and  $Q_c$  and plot on abscissae of  $\frac{V}{ND}$ . Fair curves were then drawn as nearly as might be through the points determined. The efficiency was calculated for each 0.1 or 0.05 of the  $\frac{V}{ND}$  range from values of  $T_c$  and  $Q_c$  taken from the curves. In this present report the efficiency has been calculated directly from the observations and the efficiency curve drawn through the points thus determined, serving to eliminate errors

of judgment in drawing curves of  $T_c$  and  $Q_c$ . The curves of  $T_c$ ,  $Q_c$ , and  $\rho$  are checked for consistency at values of  $\frac{V}{ND}$  varying by increments of 0.1.

Plates XXXV to XLI show the Eiffel logarithmic diagrams for the 16 models tested. In these diagrams, which may be employed in the same manner as those of Report No. 14, the metric horsepower of 75 kilogram meters per second is used.

#### DISCUSSION OF RESULTS.

*Thrust in relation to pitch ratio.*—As expected from the tests of Report No. 14 the thrust of propellers 80, 81, 82, and 83 (1.1 pitch ratio) is greater for the same value of  $\frac{V}{ND}$  than the thrust of propellers of similar form and area but of less pitch ratio.

*Thrust in relation to blade area.*—The pairs of propellers 80–81 and 82–83 show the same relations of thrust as similar pairs of Report No. 14—that is, the wide blades (81 and 83) show the lesser thrust at low slip and the greater thrust at high slip. As noted in Report No. 14, the lower thrust at low slip is probably due to the smaller dynamic pitch caused by the smaller curvature of the back of the blades.

*Thrust in relation to blade form.*—Forms 3 and 5 (curved edge entering) show less thrust than forms 4 and 6, while form 7 (straight edge entering) shows less thrust than form 8. Such differences as exist between pairs  $F_3$ – $F_4$ ,  $F_5$ – $F_6$ , and  $F_7$ – $F_8$  are believed to be due to greater tendency of one than the other propeller to warp when under load.

The differences between  $F_2$ ,  $F_3$ , and  $F_5$  and between  $F_4$  and  $F_6$  are not marked, the thrust curves being practically identical at many points.

*Thrust in relation to blade section.*—The thrust for  $S_2$  is in general intermediate between thrusts for  $S_1$  and  $S_3$ .

*Power in relation to pitch ratio.*—As was anticipated, for a given value of  $\frac{V}{ND}$  the torque and power coefficients are higher for the propellers of 1.1 pitch ratio than for those of similar form and area but of less pitch ratio.

*Power in relation to blade area.*—The pairs of propellers 80–81 and 82–83 show the same general relations of power as of thrust. The wide blades, having a less dynamic pitch, absorb less power at low slip, but at the high slips absorb the greater power. The two areas absorb the same power at a value of  $\frac{V}{ND}$  slightly less than for maximum efficiency.

*Power in relation to blade form.*—The power curves for the various forms have the same general relations as those for thrust.

*Power in relation to blade section.*—The power absorbed by propellers of blade section  $S_2$  is, in general, intermediate between powers absorbed by  $S_1$  and  $S_3$ .

*Efficiency in relation to pitch ratio.*—As was anticipated, propellers of 1.1 pitch ratio show a longer range of efficiency and a higher maximum efficiency than those of less pitch ratio.

*Efficiency in relation to blade area.*—Propeller pairs 80–81 and 82–83 show the same characteristics in efficiency curves as similar pairs in Report No. 14. The narrow blades have a greater efficiency and, having a slightly greater dynamic pitch, have a correspondingly larger range.

*Efficiency in relation to blade form.*—Such differences as exist in the efficiency of the various forms tested are not marked. Curves for  $F_2$ ,  $F_3$ , and  $F_5$  are very similar except near the maximum, where  $F_5$  has the advantage of about 1 per cent. Curves for  $F_4$  and  $F_6$  do not vary widely from the above. Curves for  $F_7$  and  $F_8$  are practically identical.

*Efficiency in relation to blade section.*—The cambered blades of  $S_2$  give efficiency curves between those for  $S_1$  and  $S_3$ , and it therefore appears that to be of advantage camber should be less than that used in  $S_3$ .

It should be noted that a table has been given showing corrected values of efficiency for propellers of Report No. 14, and that the above statements as to efficiency are made in consideration of these corrected values.

## II. TESTS OF A VARIABLE PITCH PROPELLER.

Propeller model No. 96, figure 11, was constructed with a view to determining the aerodynamic qualities of the variable pitch propeller. The blades are independent and fastened in a spherical bronze hub, as shown. By loosening the screws they may be set to any desired angle and then secured. In this propeller the blades were made of San Domingo mahogany in one piece and have the form section, etc.,  $F_1$ ,  $A_1$ ,  $S_1$ , and a constant pitch of 25.2 inches (pitch ratio, 0.7). This model is therefore like model No. 5 of Report No. 14, except for slight variation at the hub.

The model was tested in the usual manner with the following settings:

A=4° retard.	E=12° advance.
B=0° advance.	F=16° advance.
C=4° advance.	G=20° advance.
D=8° advance.	H=24° advance.

In setting A the pitch angles are 4° less than those for 0.7 pitch ratio; in setting B the blade is set at the designed pitch, and in settings C, D, E, F, G, and H the pitch angles are the amounts shown greater than those for 0.7 pitch ratio.

Results of tests are shown in Plates XIII, XIV, and XLII. The following Table IV shows the nominal and dynamic pitch and for the various settings.

TABLE IV.—Propeller model No. 96.

Setting.	Nominal pitch.	Dynamic pitch.	Mean nominal pitch ratio.	Dynamic pitch ratio.
	<i>Inches.</i>	<i>Inches.</i>		
A.....	21.2 to 17.9	23.4	0.54	0.79
B.....	21.2 to 25.2	25.2	.70	.95
C.....	29.4 to 25.2	40.7	.85	1.13
D.....	24.1 to 40.8	45.8	1.04	1.30
E.....	39.8 to 45.2	54.0	1.22	1.50
F.....	45.2 to 53.2	61.0	1.43	1.71
G.....	53 to 65	70.2	1.65	1.95
H.....	60 to 78.8	80.7	1.88	2.24

In the above table the column *nominal pitch*, except for setting B, has two values. The first is for the 7-inch radius and the second for the 16-inch radius. The column *mean nominal pitch ratio* is the mean value of nominal pitch for the 7, 10, 13, and 16 inch radii divided by the diameter.

As shown by Plates XIII, XIV, and XLII, the power absorbed by this model increases with the angle of attack for any value of  $\frac{V}{ND}$ .

For the thrust this rule does not apply. At  $\frac{V}{ND}=0.6$ , for example, there is no increase in thrust for angles of advance greater than 16°. (Setting F.)

The maximum efficiency of this propeller increases with the angle of advance until about 8° has been reached corresponding to a mean nominal pitch ratio of 1.04.

## III. TESTS OF RIGHT AND LEFT HAND PROPELLERS IN SERIES.

In order to determine the aerodynamic characteristics of two propellers (right and left hand) in series, three additional propeller models were constructed. These are numbered 97, 98, and 99. All are left hand and have form, area, and section symbols  $F_1$ ,  $A_1$ ,  $S_1$ , described in Report No. 14. No. 97 has a pitch ratio of 0.9 and is therefore like propeller No. 1 in every respect except direction of rotation. Nos. 98 and 99 have pitch ratios of 0.7 and 0.5, respectively, and are therefore duplicates to propellers No. 5 and No. 9, except as to direction of rotation.

The three sets, 1 with 97, 1 with 98, and 1 with 99, were mounted as shown in figure 12. The right-hand propeller was placed in the rear, on the shaft of the dynamometer that was located within the experiment chamber. A dynamometer (similar in operation) was built 4 feet in front of the collector end of the tunnel, the propeller shaft being extended through the collector and supported at the end by a bearing in the bronze nut which secured the right-hand propeller in

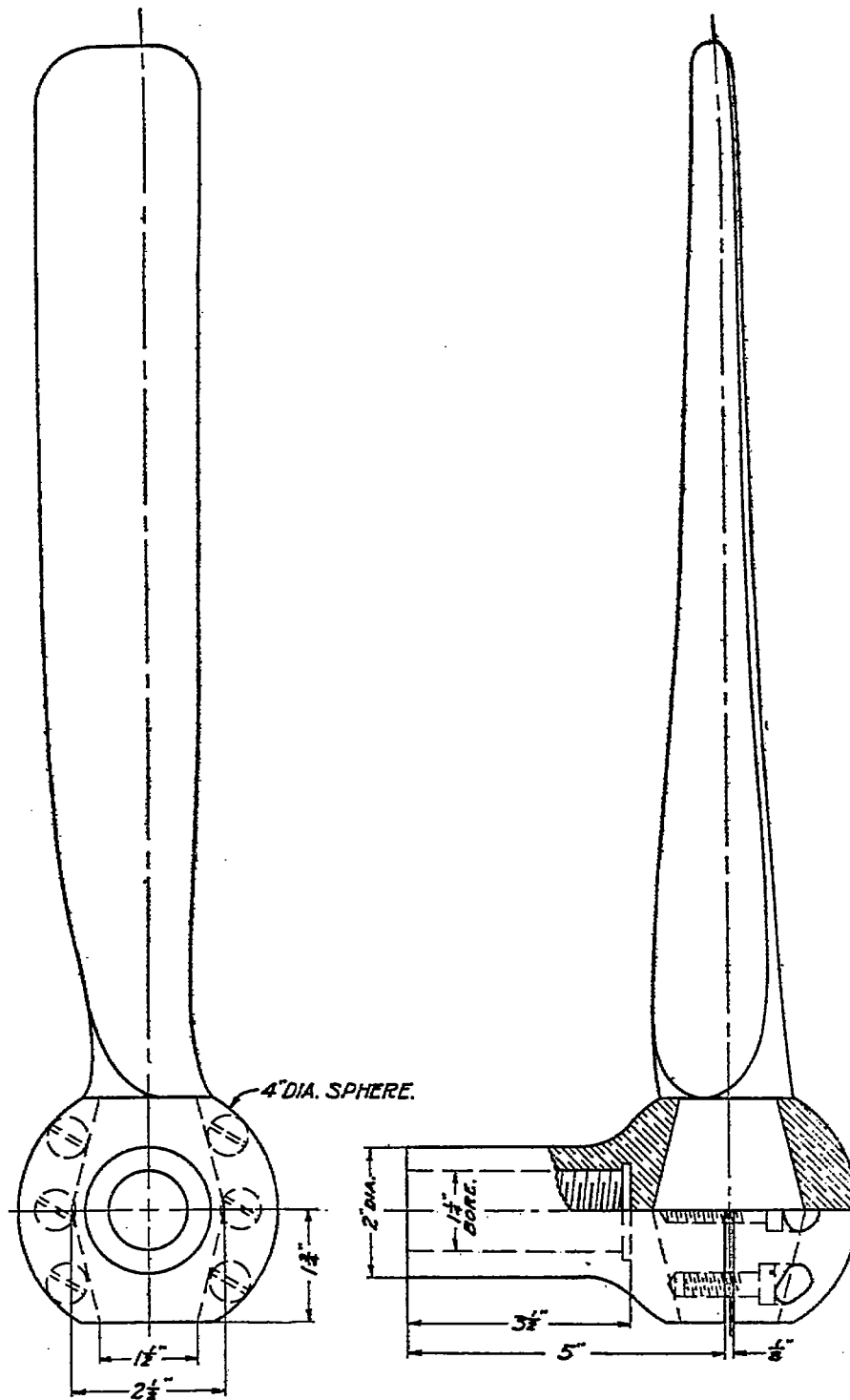


FIG. 11.

position. The left-hand propeller was mounted on the shaft of the forward dynamometer. Both propeller shafts floated freely, but the average distance between the two propellers was 6 inches.

The method of conducting tests was as follows: Both dynamometers were allowed to run until steady conditions were attained. Both dynamometers were carefully calibrated for torque and thrust. The propellers were then put in position and with a practically constant wind velocity and a constant rotative velocity of the forward (left-hand) propeller, a series of observations were made at various speeds of the rear (right-hand) propeller. The rotative velocity of the forward propeller was then changed and another series of observations made. This was continued until the range of possible utility was covered.

The observed data was reduced to thrust and torque coefficients and efficiency for each propeller and to combined efficiency for the two propellers. The coefficients and efficiencies were plotted as elevations of a surface, and the contours shown in Plates XV to XXIX were then interpolated. Plates XV to XXVI show also the torque and thrust coefficients and efficiency for propellers 1, 97, 98, and 99 when alone.

From these contours the following conclusions are drawn:

- (1) The thrust of either propeller is reduced if the other develops thrust.
- (2) The torque of the forward propeller is reduced if the rear propeller develops thrust.

(3) At low slips (the larger values of  $\frac{V}{ND}$ ) the torque of the rear propeller is reduced if the forward propeller develops thrust, while for high slips (the lower values of  $\frac{V}{ND}$ ), the torque of the rear propeller is increased if the forward propeller develops thrust.

- (4) The efficiency of either propeller is reduced if the other develops thrust.

(5) To give equal thrust or to absorb equal power at equal values of  $\frac{V}{ND}$ , two propellers (right and left in series) should have the same pitch ratio.

(6) For two like propellers delivering equal thrust or absorbing equal power, the efficiency of the combination is approximately equal to the efficiency of either at the combined thrust or power.

It may be noted that the efficiency curve for propeller No. 97 is somewhat lower than that for No. 1, although these propellers were constructed as nearly alike as possible. Measurements after tests showed No. 97 to have a slightly less pitch than No. 1. Likewise Nos. 98 and 99 have less pitch than the right hand similar ones, Nos. 5 and 9, of Report No. 14. In addition, it may be seen that the efficiency curve of No. 97 is that determined by the tests of Report No. 14 and not as determined by recent tests. The arrangement of the apparatus and time required to remove and replace propellers made it impracticable to make these tests by the improved method.

Therefore it is possible that the efficiency and torque curves shown on these diagrams are slightly in error. The relation of one curve to another is, however, believed to be substantially correct.

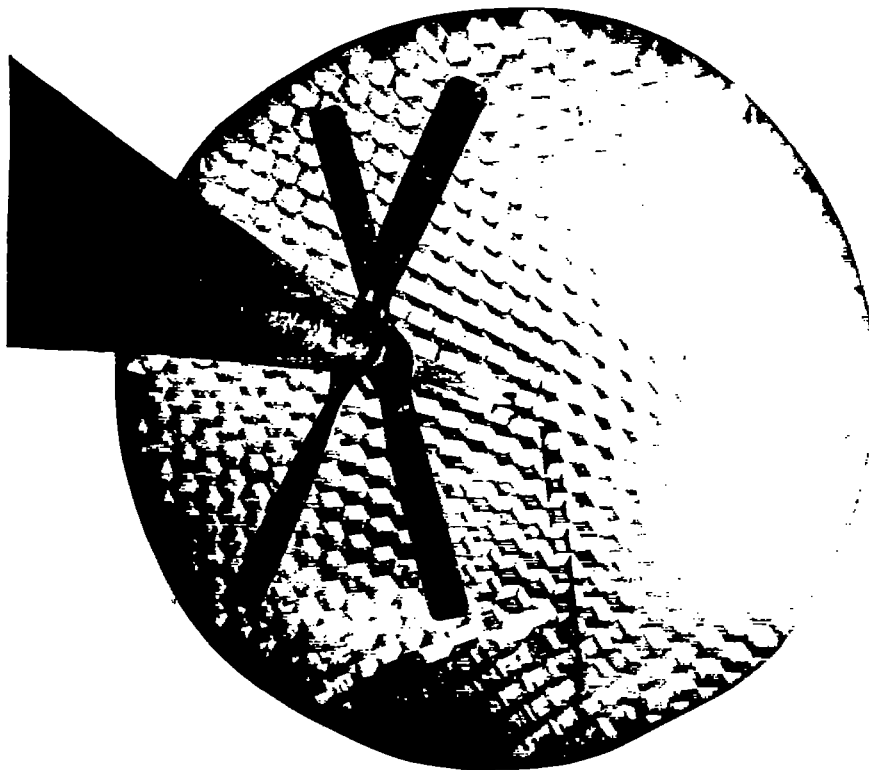
#### IV. TESTS ON MODEL PROPELLERS TO DETERMINE BRAKE EFFECT OR NEGATIVE THRUST AT NEGATIVE SLIP.

The propellers selected for these tests are numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. Their form, section, and area characteristics are fully described in Report No. 14.

The models were mounted as usual and the driving motor constrained, by means of a mechanical or an electrical brake, to run at less revolutions than required to develop a positive thrust. The wind velocity was maintained constant at about 50 miles (80 km.) per hour. A series of observations were made on each propeller at revolutions from that required for zero thrust to revolutions as low as practicable for steady conditions. The negative thrust was measured by reversing the balance arm of the thrust dynamometer.

The data obtained were reduced to coefficients in the form of the thrust coefficient  $T_c$  and plotted as shown in diagrams of Plates XXX and XXXI. It should be noted that the brake effect coefficient is plotted as a positive quantity obtained by multiplying the thrust coefficient by  $-1$ .

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FIG. 12



The diagrams show, as would be anticipated, that the narrow blades have less brake effect than the wide ones (see pairs 1-2, 3-4, 5-6, etc.), and that the brake effect of the higher pitch ratio is less than that of the lower. (Sets 1-5-9, 2-6-10, etc.) The unexpected results of this investigation are shown most clearly in the curves for propellers Nos. 10 and 12. With a constant velocity of advance, the brake effect of propeller No. 10 is at  $\frac{V}{ND} = 1.3$ , about twice the value for  $\frac{V}{ND} = 4.8$ .

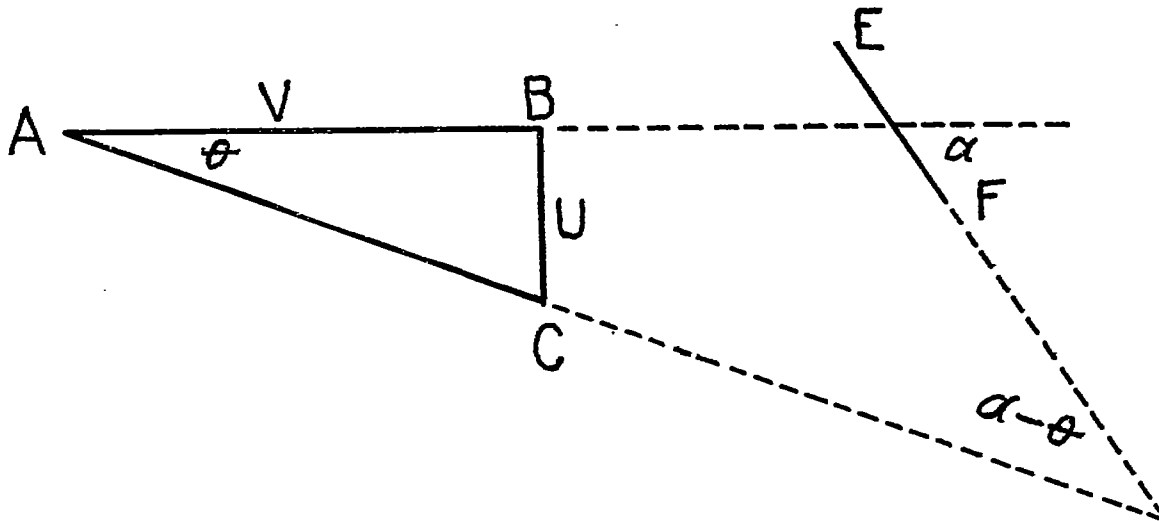


FIG. 13.

That this might have been expected may be shown by the following analysis:

Let  $EF$ , figure 13, represent an element of the propeller surface.

$AB = V$ , the constant forward velocity

$BC = U$ , the variable circumferential velocity

$AC$  is then the relative velocity of wind and element.

$\alpha = 90^\circ$ —pitch angle.

$(\alpha - \theta)$  is then the relative direction of the wind and elementary surface.

From Duchemin's formula, verified by Langley (see experiments in aerodynamics by S. P. Langley, Smithsonian Institution, 1891) the normal pressure on the element, and hence the thrust component, is equal to  $K \frac{2 \sin(\alpha - \theta)}{1 + \sin^2(\alpha - \theta)}$ .

In the above formula, the velocity is presumed to be constant.

The velocity of the element, however, with constant velocity of advance, varies as  $\sec \theta$ . Merging the coefficient 2 with the constant  $K$  and assuming that pressure varies as velocity squared, we may write; Thrust =  $K \frac{\sin(\alpha - \theta) \sec^2 \theta}{1 + \sin^2(\alpha - \theta)}$ .

Assigning to  $\alpha$  values of  $70^\circ$  and  $85^\circ$  (corresponding to pitch angles of  $20^\circ$  and  $5^\circ$ ) and to  $\theta$  various values, a series of values for thrust may be determined. These, if plotted on abscissæ of  $\cot \theta$ , give curves as shown in figure 14, which have the same general shape as curves shown on Plates XXX and XXXI. For the above analysis the author is indebted to Prof. L. M. Hoskins, of Stanford University.

The curves shown for propellers Nos. 3 and 12 may also be transformed into curves similar to one derived by the Duchemin formula as follows:

$$\cot \theta = \frac{V}{U} = \frac{V}{\pi ND}$$

By Duchemin's formula  $P = 0$  when  $\alpha - \theta = 0$  or when  $\alpha = \theta$ . But from plate XXXI, thrust (and consequently  $P$ ) is equal to zero when  $\frac{V}{ND}$  is .068 for propeller No. 12 and 1.2 for propeller No. 3.

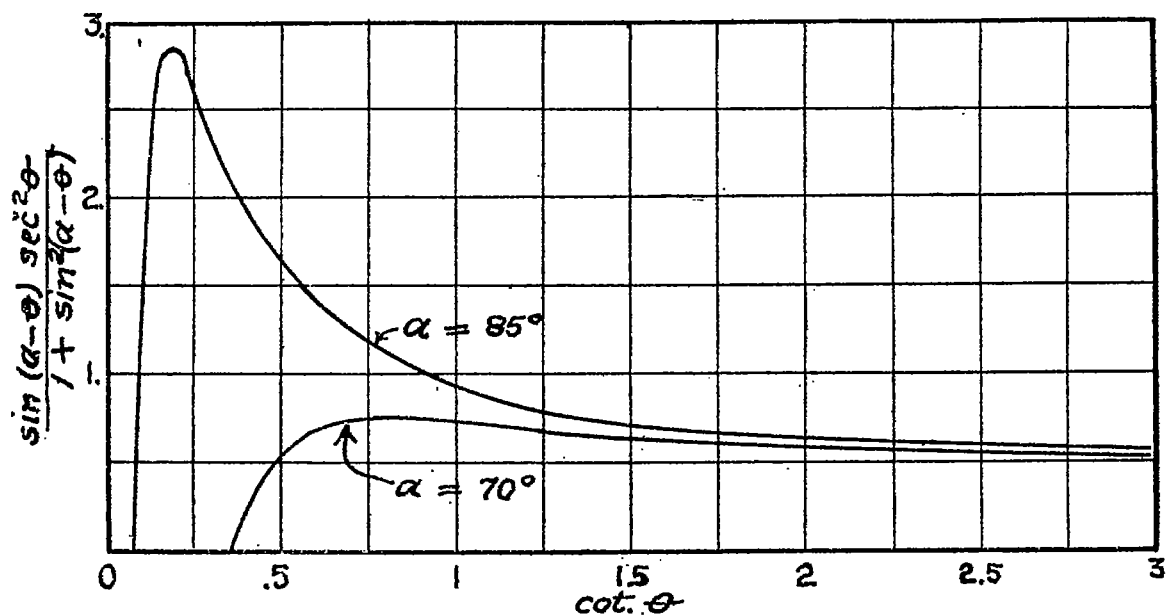


FIG. 14.

The equivalent  $\alpha$  is then

for propeller No. 12,  $\cot^{-1} \frac{0.68}{\pi}$ , or  $77^\circ 45'$

and for propeller No. 3,  $\cot^{-1} \frac{1.2}{\pi}$ , or  $69^\circ 6'$ .

If ordinates of curves for propellers 12 and 3 are divided by  $\sec^2 \left( \cot^{-1} \frac{V}{\pi ND} \right)$ , the quotients multiplied by a constant and plotted on abscissae of  $77^\circ 45' - \cot^{-1} \frac{V}{\pi ND}$  (for propeller 12) and  $69^\circ 6' - \cot^{-1} \frac{V}{\pi ND}$  (for propeller 3) the points will lie as shown in figure 15. These tests therefore serve to confirm the Duchemin formula.

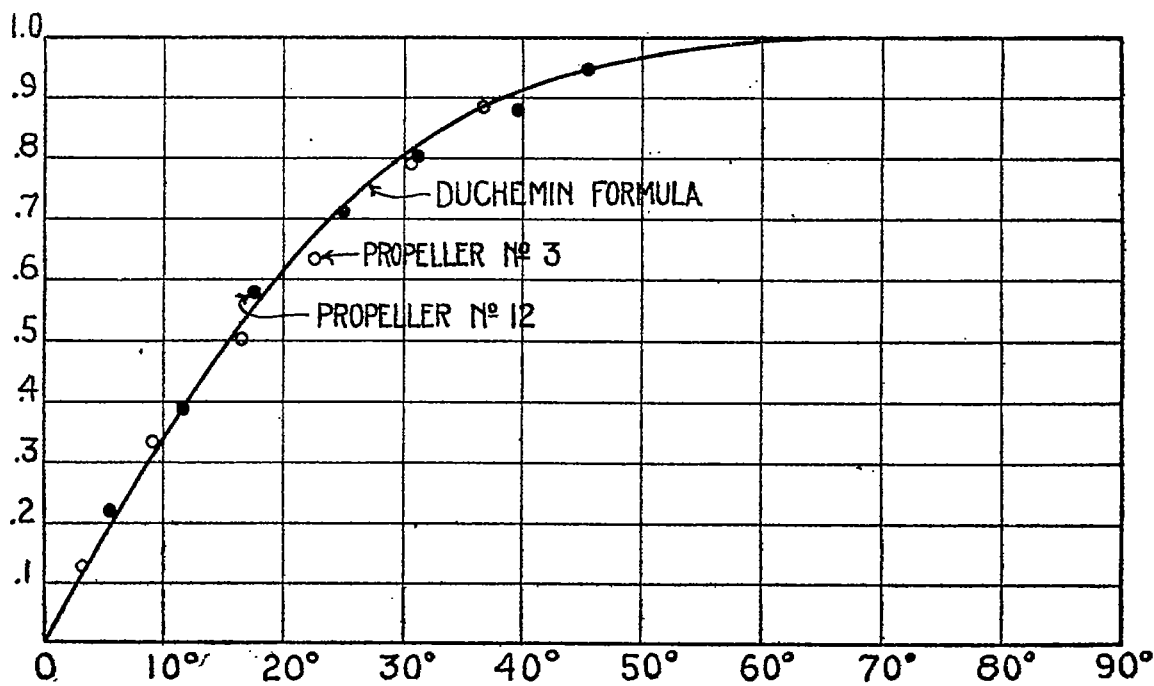


FIG. 15.

## V. STANDING THRUST AND POWER TESTS.

The 51 propellers of Report No. 14 and the 16 propellers of this present report were tested to determine coefficients of standing thrust and power.

This investigation was necessarily carried on in the wind tunnel. Preliminary tests were therefore conducted to determine the effect of the velocity generated throughout the tunnel by the action of the model alone, with a view to eliminating such effect if possible. It was found that if the experiment chamber were closed, except for the openings to collector and diffuser, the velocity generated by the model was sufficient to make results of tests inconsistent. After several trials the following conditions were established as closely approximating operation in free air. Doors and windows of the experiment chamber were removed, and the diffuser partially closed by a damper, so that the stream driven backward by the model just passed out through the diffuser without forming eddies and backward currents in the experiment chamber.

All propellers were tested throughout the practicable range of revolutions. The observations were reduced to coefficients as follows:

$$\frac{\text{Thrust}}{\Delta N^2 D^4} \text{ and } \frac{\text{Power}}{\Delta N^3 D^5}$$

These coefficients were plotted as ordinates on abscissae of  $ND$ , as shown in Plates XXXII, XXXIII, and XXXIV. The diagrams are typical of the 67 propellers tested. For the most part the coefficients  $\frac{\text{Thrust}}{\Delta N^2 D^4}$  and  $\frac{\text{Power}}{\Delta N^3 D^5}$  plot as straight lines nearly parallel to the axis of  $ND$ , showing the coefficients to be practically constant.

The data for the 67 propellers is given in the following Table V.

TABLE V.—Standing thrust and power.

Propeller number.	$ND$	$\frac{\text{Thrust}}{\Delta N^2 D^4}$	$\frac{\text{Power}}{\Delta N^3 D^5}$	$\frac{\text{Thrust}}{\text{Power.}}$
1.....	10	0.0140	0.00878	0.1592
	20	.0153	.00859	.0891
	30	.0153	.00881	.0880
	40	.0153	.00905	.0428
2.....	10	.0190	.00917	.2071
	20	.0194	.01087	.0218
	30	.0199	.01108	.0898
	40	.0202	.01138	.0458
3.....	10	.0128	.00829	.1544
	20	.0163	.00820	.0994
	30	.0168	.00822	.0973
	40	.0160	.00840	.0477
4.....	10	.0192	.01006	.1910
	20	.0195	.01045	.0832
	30	.0198	.01040	.0828
	40	.0195	.01035	.0471
5.....	10	.0118	.00628	.1880
	20	.0140	.00698	.1008
	30	.0153	.00731	.0997
	40	.0162	.00768	.0827
6.....	10	.0160	.00742	.2155
	20	.0162	.00792	.1025
	30	.0167	.00802	.0893
	40	.0170	.00808	.0817
7.....	10	.0130	.00667	.1985
	20	.0145	.00661	.1068
	30	.0147	.00650	.0768
	40	.0149	.00662	.0863
8.....	10	.0151	.00690	.0438
	20	.0160	.00715	.2235
	30	.0160	.00738	.1085
	40	.0163	.00754	.0720
9.....	10	.0164	.00769	.0683
	20	.0102	.00482	.2255
	30	.0111	.00482	.1160
	40	.0117	.00482	.0809
	10	.0120	.00482	.0622
	20	.0128	.00485	.0606

TABLE V.—Standing thrust and power—Continued.

Propeller number.	$ND$	$\frac{\text{Thrust}}{\Delta N^2 D^4}$	$\frac{\text{Power}}{\Delta N^3 D^5}$	$\frac{\text{Thrust}}{\text{Power.}}$
10.....	10	0.0122	0.00509	0.2398
	20	.0124	.00505	.1226
	30	.0127	.00519	.0816
	40	.0129	.00529	.0610
11.....	10	.0130	.00535	.0486
	20	.0110	.00465	.2366
	30	.0115	.00470	.1225
	40	.0116	.00470	.0822
12.....	10	.0118	.00470	.0827
	20	.0120	.00470	.0610
	30	.0122	.00496	.2460
	40	.0122	.00496	.1230
13.....	10	.0122	.00492	.0825
	20	.0122	.00490	.0823
	30	.0122	.00489	.0800
	40	.0122	.00489	.0800
14.....	10	.0126	.00887	.1453
	20	.0149	.00887	.0840
	30	.0164	.00873	.0827
	40	.0176	.00860	.0518
15.....	10	.0136	.01023	.1914
	20	.0195	.01072	.0910
	30	.0197	.01123	.0583
	40	.0198	.01149	.0430
16.....	10	.0154	.00830	.1855
	20	.0168	.00813	.1002
	30	.0166	.00832	.0664
	40	.0168	.00848	.0495
17.....	10	.0188	.00943	.1995
	20	.0188	.01000	.0940
	30	.0188	.01000	.0627
	40	.0188	.01000	.0470
18.....	10	.0112	.00609	.1838
	20	.0125	.00646	.0967
	30	.0140	.00676	.0690
	40	.0151	.00668	.0541
	10	.0164	.00742	.2210
	20	.0162	.00747	.1085
	30	.0164	.00759	.0694
	40	.0165	.00818	.0505

TABLE V.—Standing thrust and power—Continued.

Propeller number.	N D	Thrust. $\Delta N D^4$	Power. $\Delta N D^5$	Thrust. Power.
19.....	10	0.0126	0.00315	0.2047
	20	.0138	.00583	.1184
	30	.0138	.00594	.0774
	40	.0137	.00601	.0570
20.....	10	.0163	.00741	.2300
	20	.0161	.00763	.1047
	30	.0163	.00783	.0692
	40	.0164	.00800	.0523
21.....	10	.0102	.00402	.2536
	20	.0110	.00444	.1238
	30	.0114	.00465	.0817
	40	.0117	.00480	.0609
	50	.0120	.00490	.0490
22.....	10	.0124	.00478	.2600
	20	.0123	.00510	.1205
	30	.0126	.00527	.0797
	40	.0128	.00540	.0593
	50	.0129	.00550	.0469
23.....	10	.0110	.00416	.2650
	20	.0112	.00455	.1230
	30	.0112	.00444	.0840
	40	.0112	.00435	.0644
	50	.0111	.00428	.0520
24.....	10	.0120	.00452	.2653
	20	.0120	.00436	.1237
	30	.0120	.00438	.0820
	40	.0121	.00431	.0626
	50	.0121	.00434	.0490
25.....	10	.0154	.01067	.1443
	20	.0176	.01072	.0821
	30	.0179	.01103	.0541
	40	.0181	.01124	.0403
26.....	10	.0204	.01130	.1805
	20	.0206	.01183	.0871
	30	.0209	.01221	.0670
	40	.0212	.01250	.0424
27.....	10	.0160	.00967	.1655
	20	.0177	.00950	.0932
	30	.0180	.00950	.0632
	40	.0182	.00950	.0479
28.....	10	.0204	.01067	.1912
	20	.0205	.01135	.0908
	30	.0208	.01164	.0589
	40	.0208	.01192	.0432
29.....	10	.0139	.00779	.1785
	20	.0162	.00853	.0950
	30	.0179	.00933	.0638
	40	.0192	.00985	.0487
30.....	10	.0173	.00842	.2055
	20	.0179	.00893	.1001
	30	.0183	.00935	.0652
	40	.0187	.00966	.0484
31.....	10	.0144	.00748	.1926
	20	.0160	.00782	.1024
	30	.0166	.00810	.0683
	40	.0170	.00832	.0512
32.....	10	.0170	.00779	.2132
	20	.0175	.00853	.1020
	30	.0182	.00893	.0678
	40	.0187	.00925	.0506
33.....	10	.0120	.00497	.2414
	20	.0131	.00531	.1123
	30	.0139	.00521	.0745
	40	.0145	.00553	.0556
	50	.0150	.00579	.0444
34.....	10	.0125	.00490	.2551
	20	.0132	.00554	.1190
	30	.0137	.00586	.0779
	40	.0140	.00608	.0575
	50	.0143	.00625	.0458
35.....	10	.0130	.00521	.2494
	20	.0132	.00557	.1185
	30	.0134	.00565	.0790
	40	.0135	.00578	.0589
	50	.0136	.00578	.0470
36.....	10	.0134	.00534	.2610
	20	.0138	.00588	.1174
	30	.0142	.00619	.0764
	40	.0145	.00644	.0562
	50	.0148	.00663	.0446

TABLE V.—Standing thrust and power—Continued.

Propeller number.	N D	Thrust. $\Delta N D^4$	Power. $\Delta N D^5$	Thrust. Power.
37.....	10	0.0144	0.01005	0.1434
	20	.0163	.01189	.0705
	30	.0184	.01262	.0487
	40	.0197	.01311	.0376
38.....	10	.0208	.01094	.1883
	20	.0210	.01182	.0890
	30	.0218	.01271	.0573
	40	.0224	.01342	.0413
39.....	10	.0164	.00955	.1718
	20	.0176	.00922	.0955
	30	.0178	.00940	.0614
	40	.0176	.00961	.0463
40.....	10	.0200	.01081	.1849
	20	.0202	.01109	.0910
	30	.0206	.01131	.0608
	40	.0209	.01148	.0454
41.....	10	.0133	.00716	.1857
	20	.0157	.00810	.0968
	30	.0174	.00916	.0633
	40	.0188	.00993	.0470
42.....	10	.0170	.00811	.2097
	20	.0185	.00898	.1029
	30	.0190	.00956	.0662
	40	.0194	.01001	.0484
43.....	10	.0142	.00716	.1983
	20	.0157	.00748	.1056
	30	.0163	.00776	.0700
	40	.0168	.00797	.0527
44.....	10	.0170	.00754	.2254
	20	.0170	.00819	.1037
	30	.0176	.00866	.0678
	40	.0180	.00898	.0501
45.....	10	.0114	.00527	.2163
	20	.0126	.00562	.1120
	30	.0132	.00582	.0756
	40	.0137	.00597	.0578
	50	.0141	.00606	.0465
46.....	10	.0134	.00540	.2482
	20	.0141	.00612	.1151
	30	.0148	.00658	.0750
	40	.0153	.00697	.0549
47.....	10	.0120	.00430	.2450
	20	.0124	.00502	.1235
	30	.0127	.00513	.0823
	40	.0129	.00522	.0618
	50	.0130	.00528	.0493
48.....	10	.0136	.00559	.2432
	20	.0134	.00563	.1180
	30	.0135	.00569	.0791
	40	.0136	.00570	.0597
	50	.0137	.00570	.0481
49.....	10	.0122	.00535	.1921
	20	.0133	.00532	.1053
	30	.0139	.00532	.0734
	40	.0143	.00532	.0566
50.....	10	.0122	.00641	.1905
	20	.0139	.00622	.1117
	30	.0141	.00628	.0743
	40	.0143	.00632	.0566
51.....	10	.0134	.00641	.2080
	20	.0141	.00625	.1128
	30	.0141	.00642	.0732
80.....	10	.0148	.00980	.1511
	20	.0150	.01034	.0726
	30	.0151	.01095	.0459
	40	.0152	.01155	.0329
81.....	10	.0194	.01300	.1493
	20	.0198	.01322	.0750
	30	.0204	.01404	.0456
	40	.0208	.01469	.0354
82.....	10	.0152	.00917	.1658
	20	.01490	.00976	.0764
	30	.01433	.00998	.0495
	40	.01390	.01021	.0362
83.....	10	.01919	.01212	.1584
	20	.01902	.01237	.0776
	30	.01893	.01248	.0505
	40	.01881	.01271	.0370

TABLE V.—*Standing thrust and power—Continued.*

Propeller number.	$ND$	Thrust. $\Delta ND^4$	Power. $\Delta ND^4$	Thrust. Power.
84.....	10	0.01143	0.00553	0.1750
	20	.01384	.00590	.1185
	30	.01401	.00596	.0782
	40	.01418	.00609	.0582
	50	.01427	.00618	.0482
85.....	10	.0100	.00672	.1488
	20	.0138	.00616	.1115
	30	.0142	.00640	.0740
	40	.0145	.00680	.0550
	50	.0147	.00676	.0435
86.....	10	.0116	.00653	.1776
	20	.0135	.00668	.1188
	30	.0138	.00689	.0781
	40	.0141	.00696	.0581
	50	.0143	.00620	.0441
87.....	10	.0102	.00641	.1592
	20	.0139	.00614	.1132
	30	.0143	.00640	.0744
	40	.0145	.00680	.0554
	50	.0149	.00674	.0432
88.....	10	.0100	.00603	.1658
	20	.0128	.00572	.1118
	30	.0128	.00582	.0733
	40	.0128	.00590	.0542
	50	.0128	.00596	.0429
89.....	10	.0100	.00590	.1695
	20	.0127	.00609	.1042
	30	.0133	.00620	.0716
	40	.0137	.00628	.0545
	50	.0140	.00633	.0445

TABLE V.—*Standing thrust and power—Continued.*

Propeller number.	$ND$	Thrust. $\Delta ND^4$	Power. $\Delta ND^4$	Thrust. Power.
90.....	10	0.0114	0.00453	0.2517
	20	.0119	.00502	.1185
	30	.0122	.00502	.0810
	40	.0125	.00502	.0622
	50	.0127	.00502	.0506
91.....	10	.0126	.00508	.2480
	20	.0128	.00523	.1223
	30	.0131	.00531	.0823
	40	.0133	.00538	.0618
	50	.0135	.00544	.0496
92.....	10	.0137	.00716	.1913
	20	.0152	.00722	.1052
	30	.0158	.00726	.0725
	40	.0162	.00729	.0555
93.....	10	.0162	.00768	.2166
	20	.0163	.00766	.1064
	30	.0163	.00792	.0766
	40	.0172	.00810	.0531
94.....	10	.0149	.00892	.1671
	20	.0169	.00865	.0977
	30	.0169	.00865	.0652
	40	.0169	.00865	.0499
95.....	10	.0196	.01018	.1926
	20	.0196	.01051	.0932
	30	.0198	.01063	.0617
	40	.0199	.01093	.0455

In the above table power is in kilogram meters per second, and other units are metric as previously stated. Such variation as is shown in the thrust or power coefficient of any one propeller is believed to be due partly to warping of the blades when under load, and partly to inaccuracy of observations. The latter applies to values of  $ND$  below 20. Tests show that the blades when under maximum load (about 30 kilograms) may bend  $\frac{1}{8}$  inch (1.5 centimeters) at the tip which may change the tip pitch angle 2 degrees.

Regarding  $\frac{\text{Thrust}}{\text{Power}}$  as a measure of efficiency in producing standing thrust, it may be seen that it varies, between  $ND=20$  and  $ND=50$ , inversely as  $ND$  for any one propeller. For  $ND$  less than 20 the ratio is sometimes smaller and sometimes greater than would be derived by the application of this rule.

Propellers of the smaller pitch have the larger ratios,  $\frac{\text{Thrust}}{\text{Power}}$ .

Form 2 appears superior to other forms.

The wide blades ( $A_2$ ) have higher ratios  $\left(\frac{\text{Thrust}}{\text{Power}}\right)$  than the narrow ones ( $A_1$ ) at low values of  $ND$ , but at high values of  $ND$  the reverse is often true.

There is little difference between propellers of uniform and expanding pitch.

Between  $ND=20$  and  $ND=50$ , the noncambered blades give higher  $\frac{\text{Thrust}}{\text{Power}}$  than the cambered blades of the same form and section.

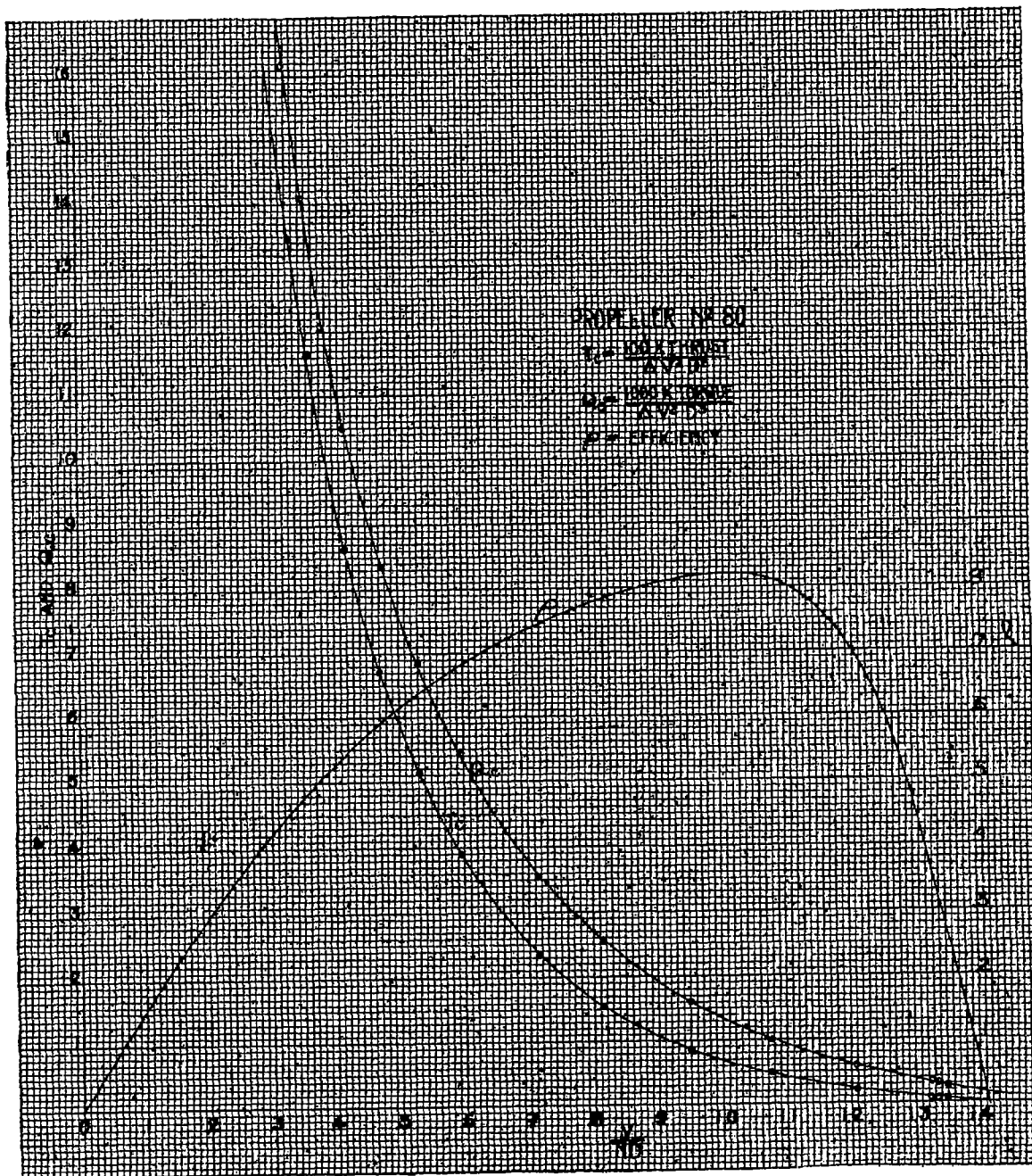


PLATE II.

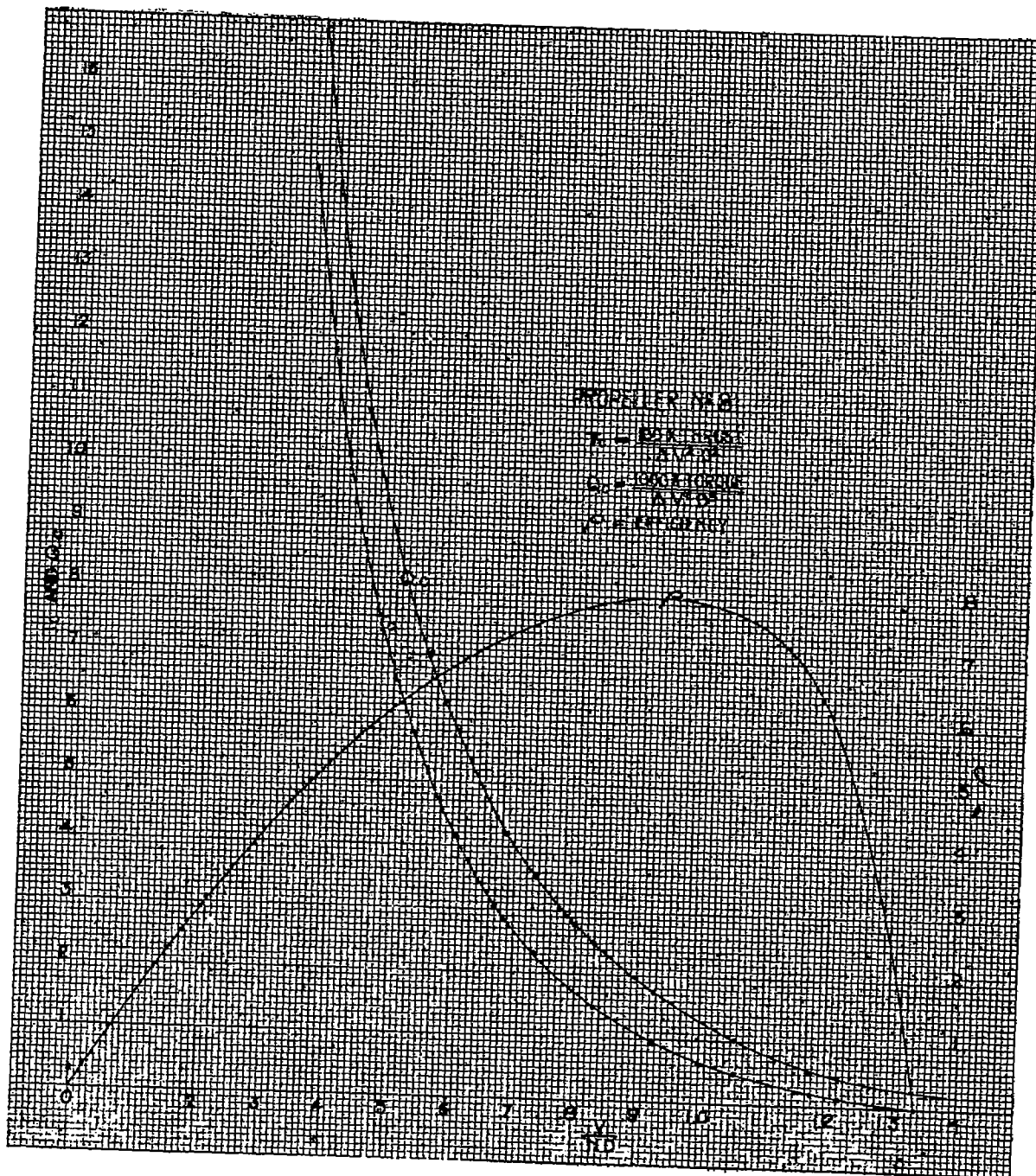


PLATE II.

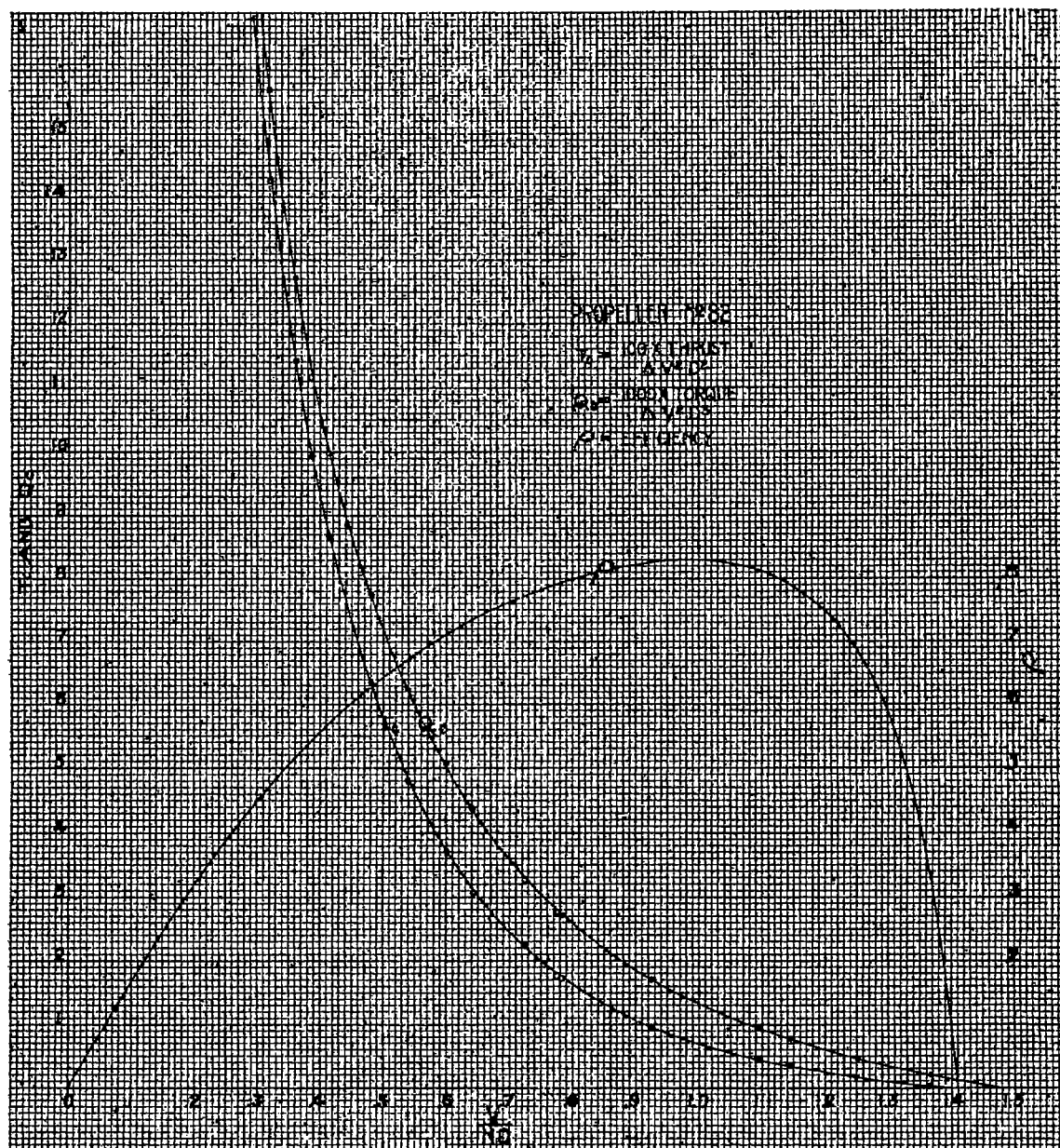


PLATE III.



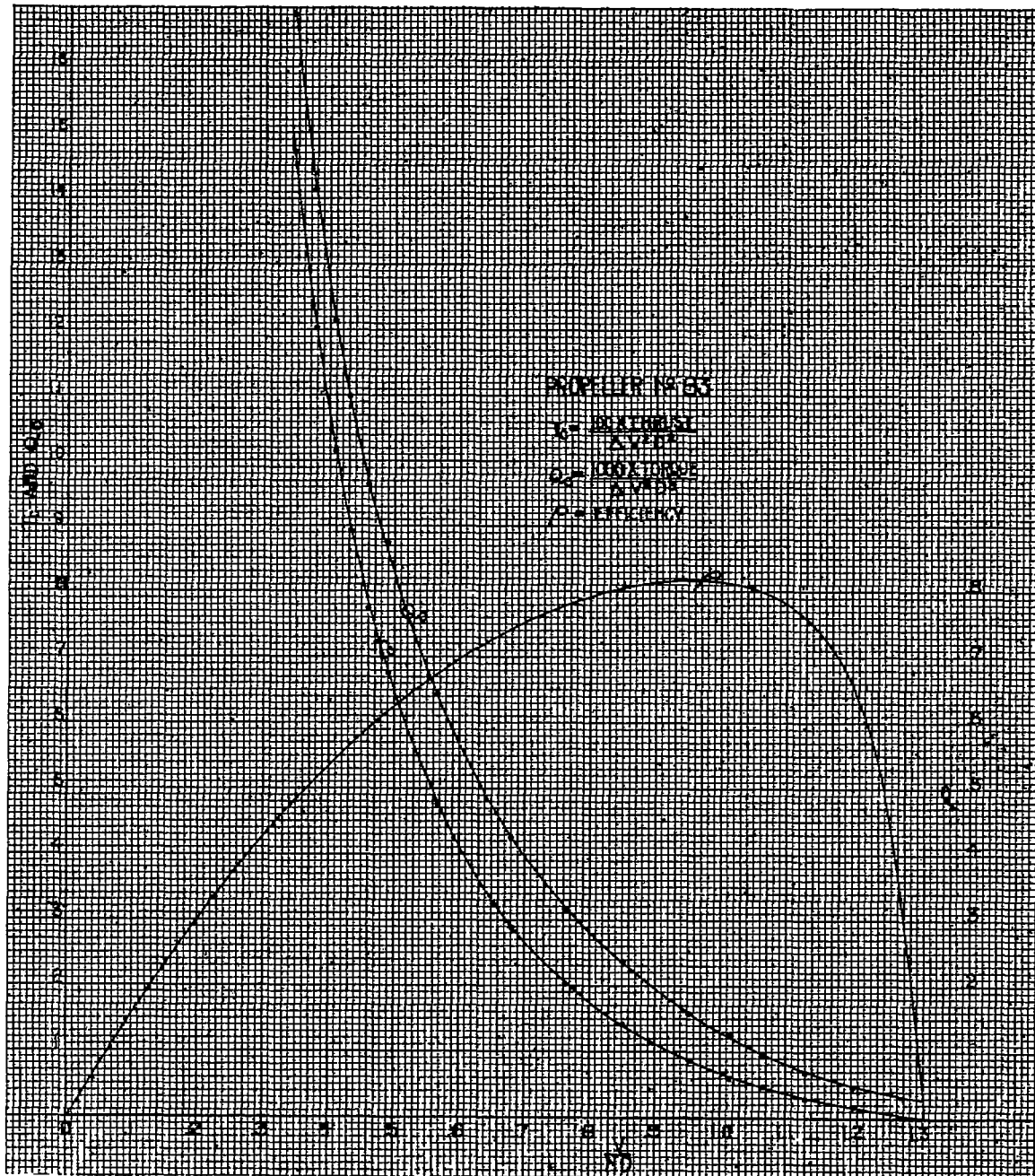


PLATE IV.

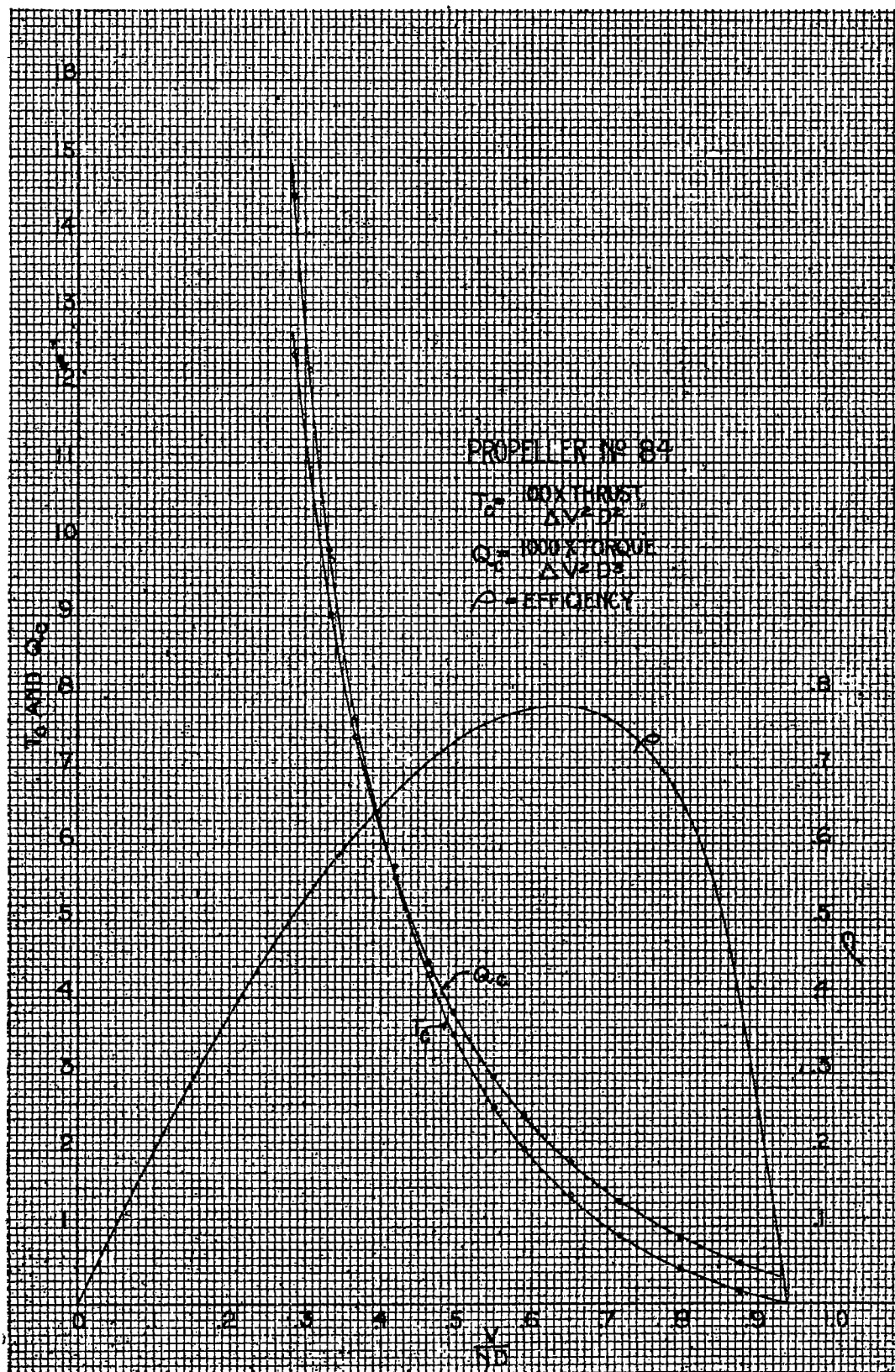


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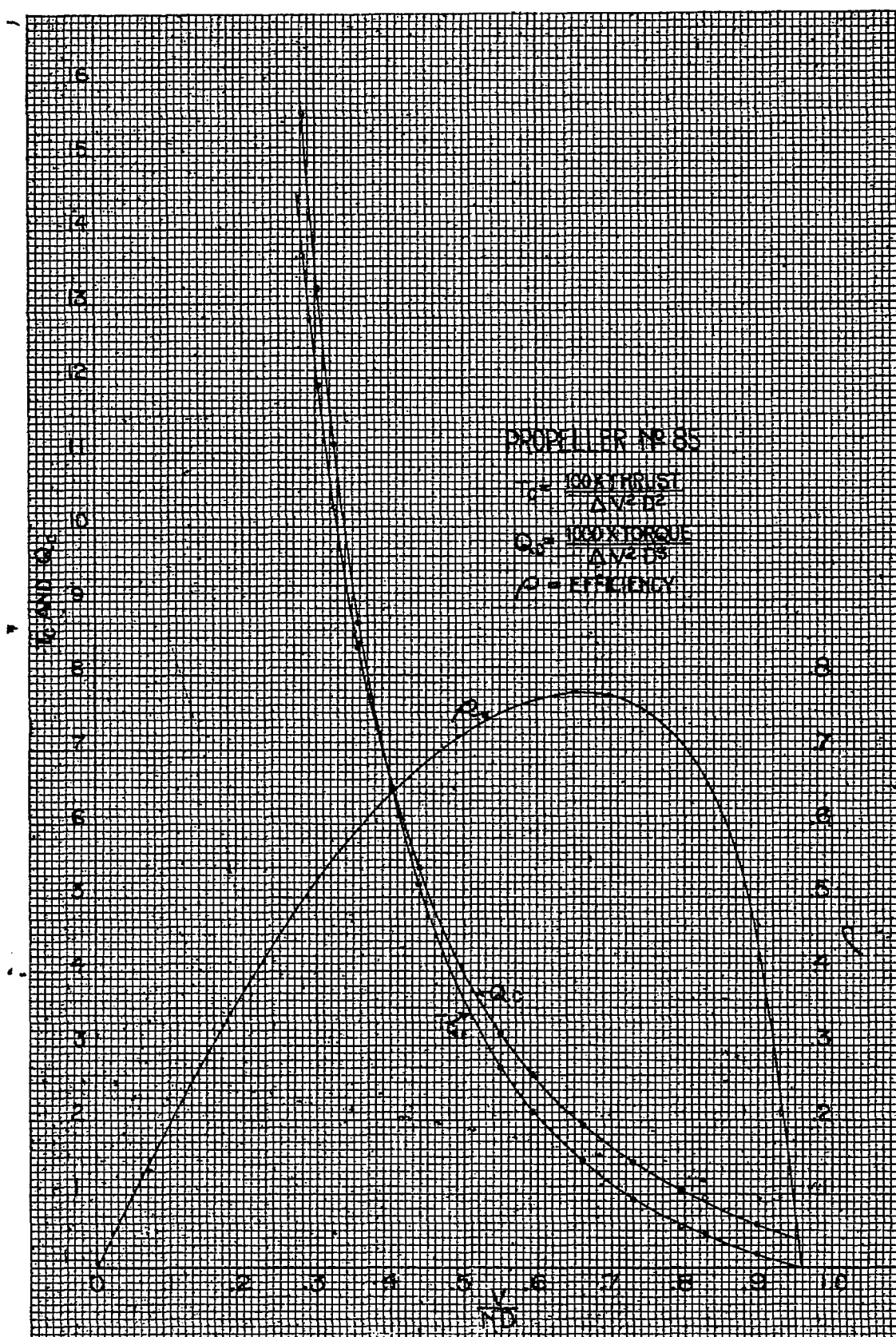


PLATE VI.

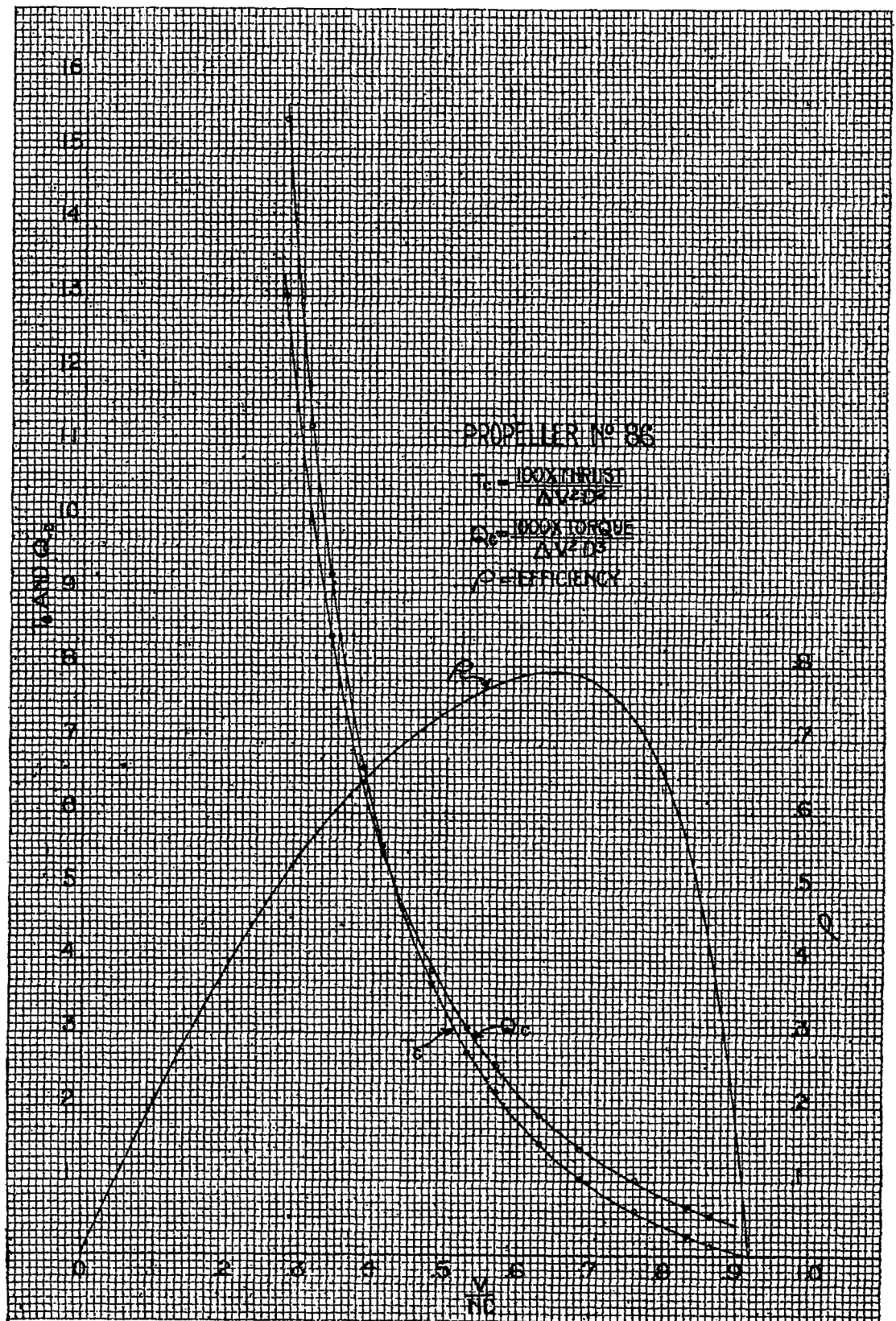


PLATE VII.

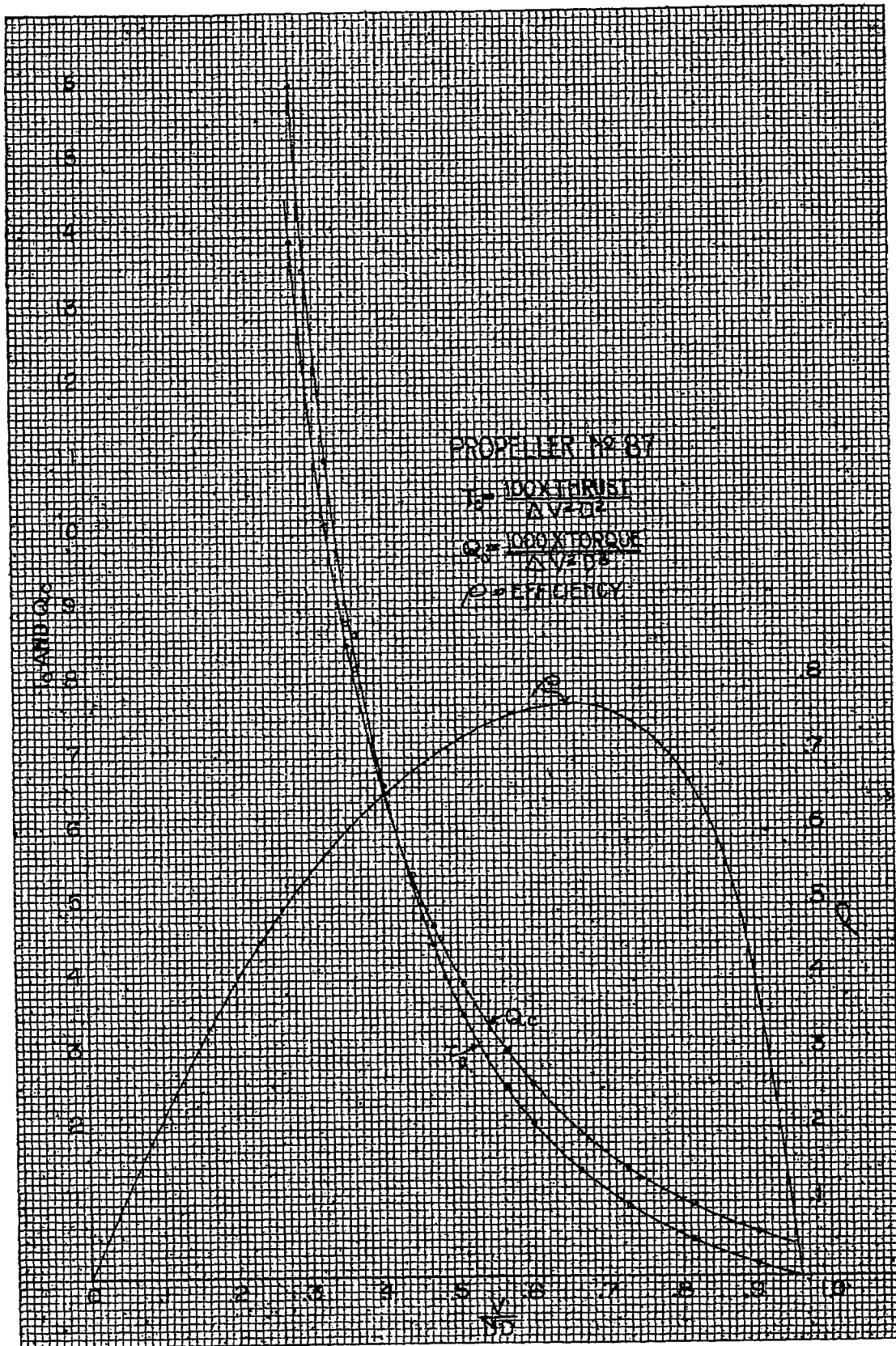


PLATE VIII.



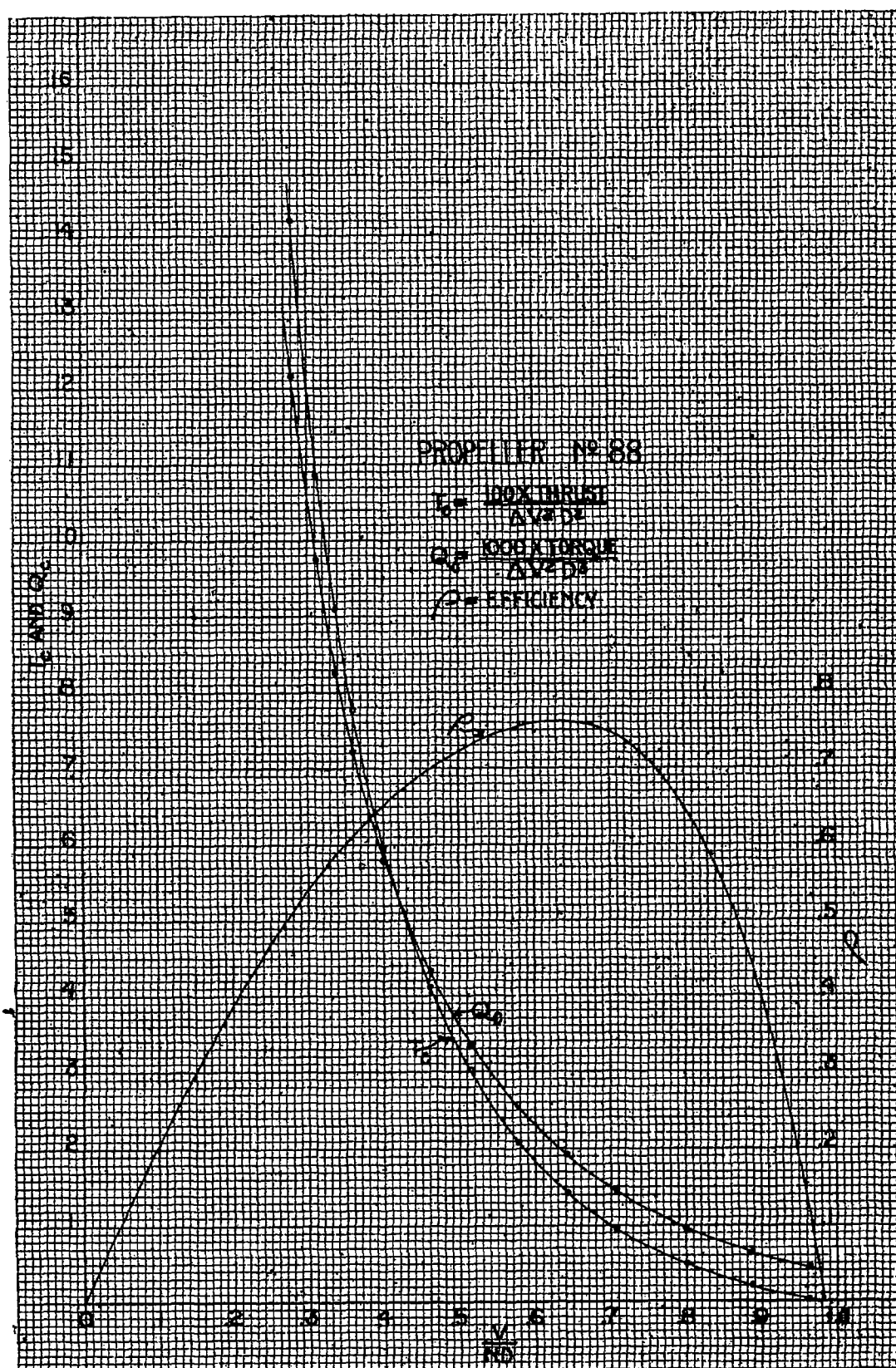


PLATE IX.

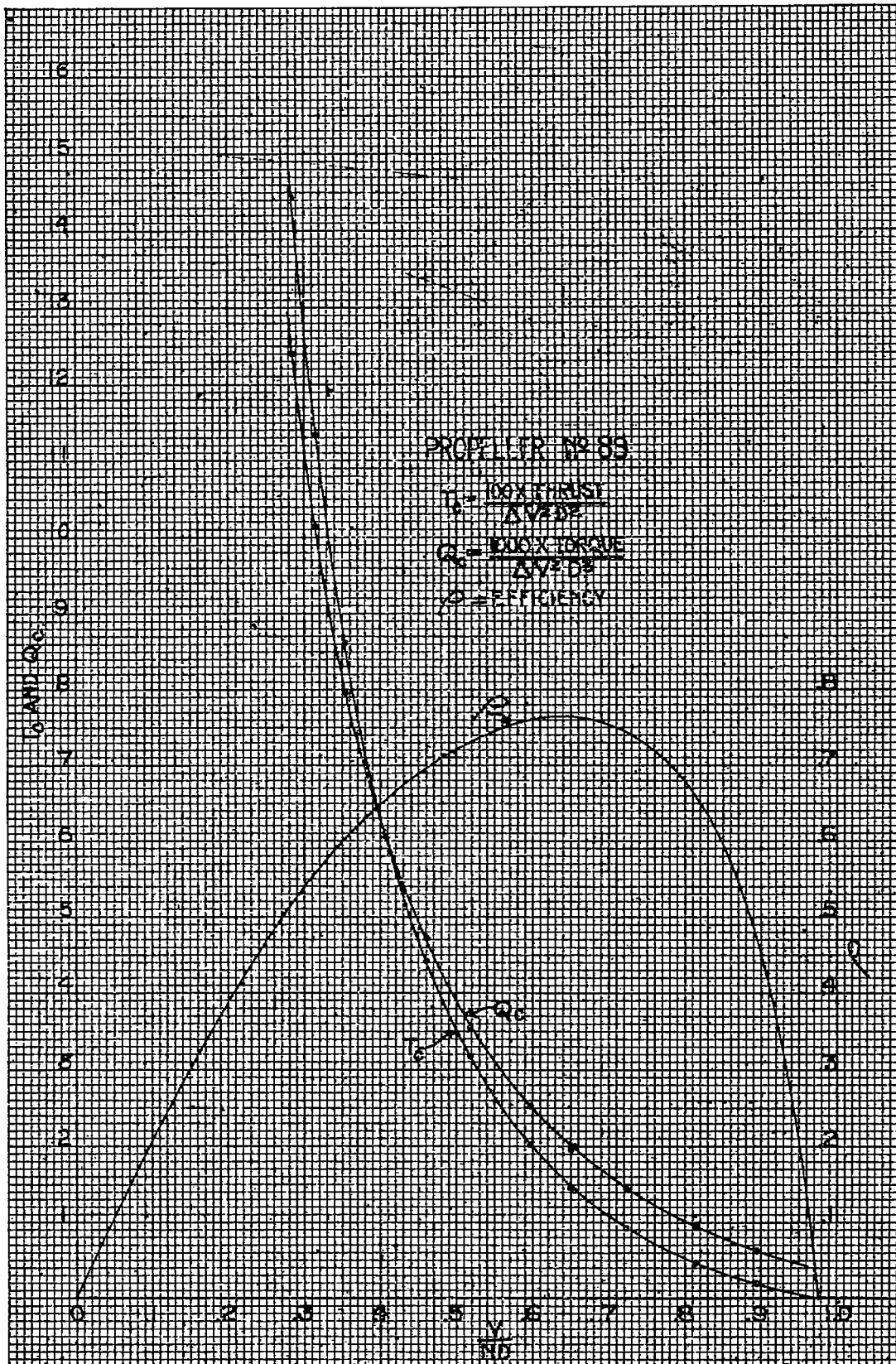
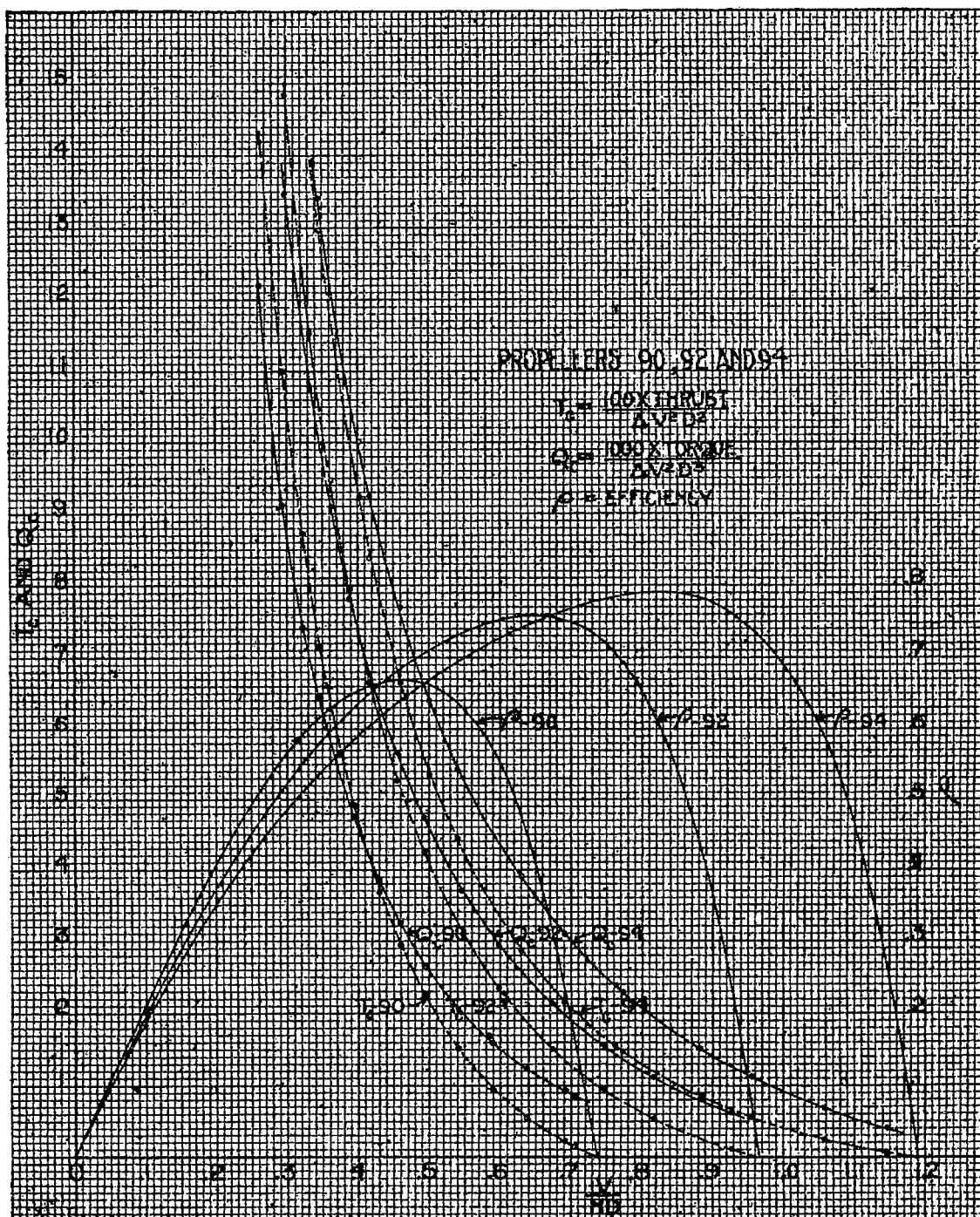


PLATE X.





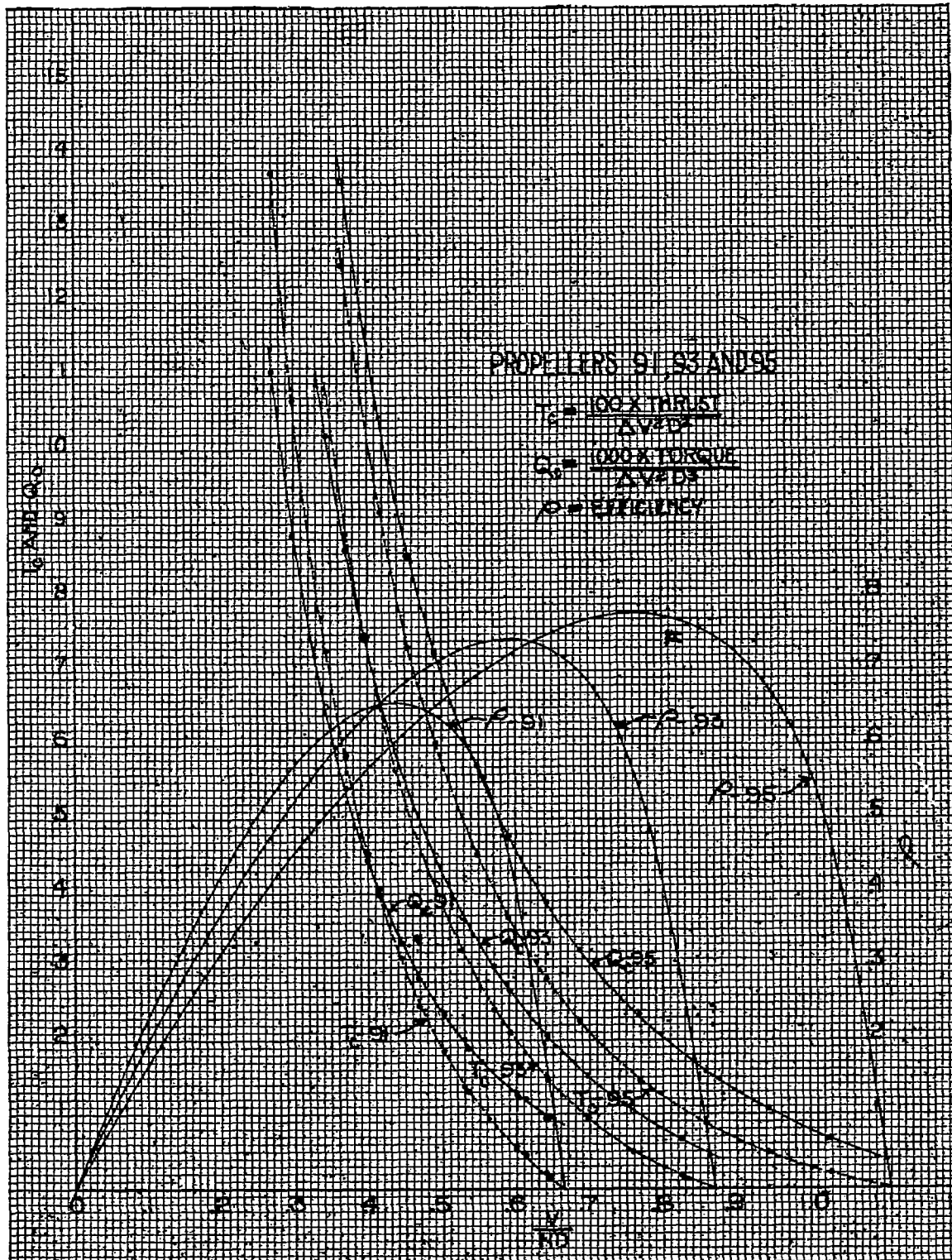
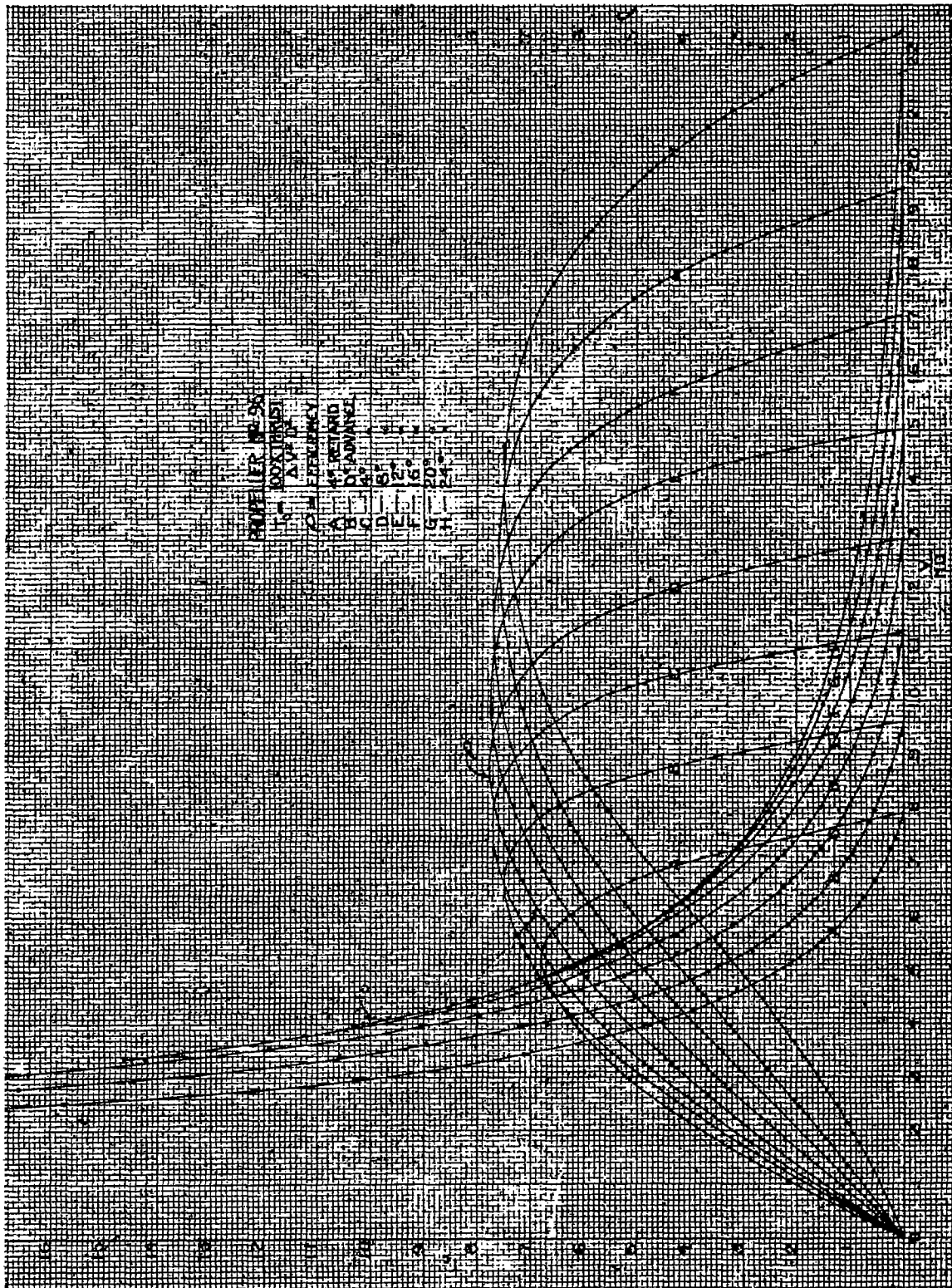


PLATE XII.



## PLATE XIII.

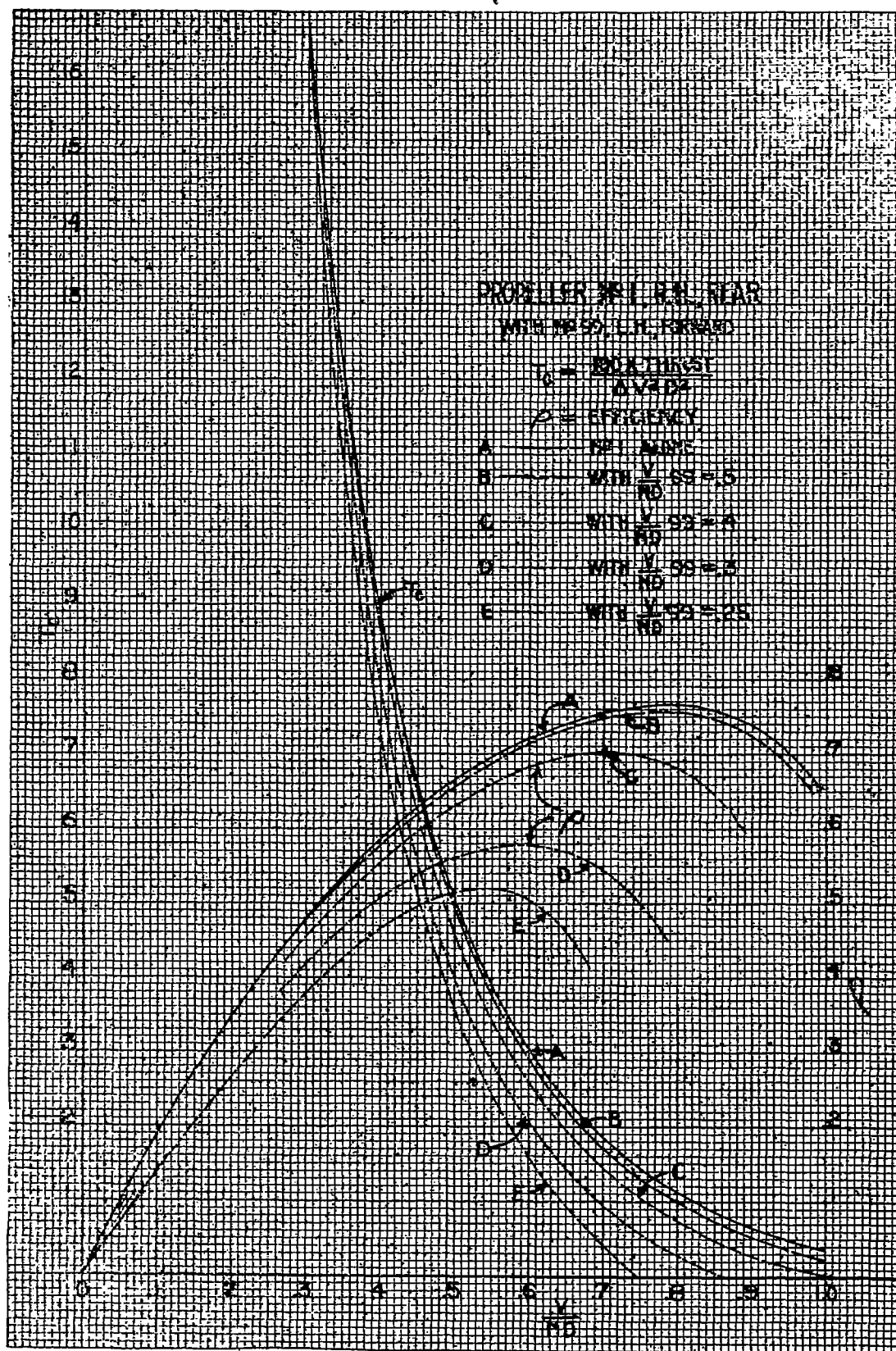


PLATE XIV.





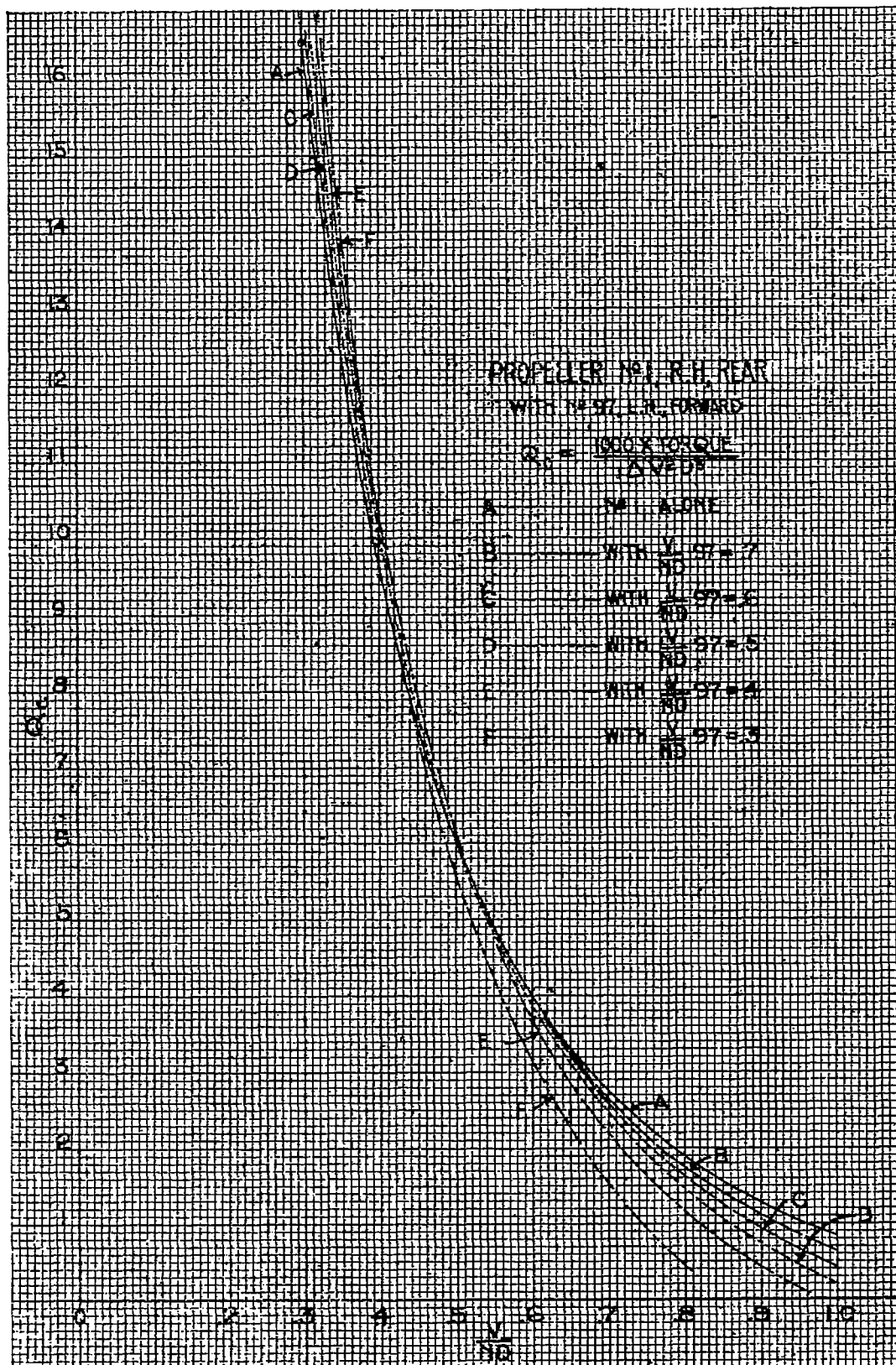


PLATE XVI.

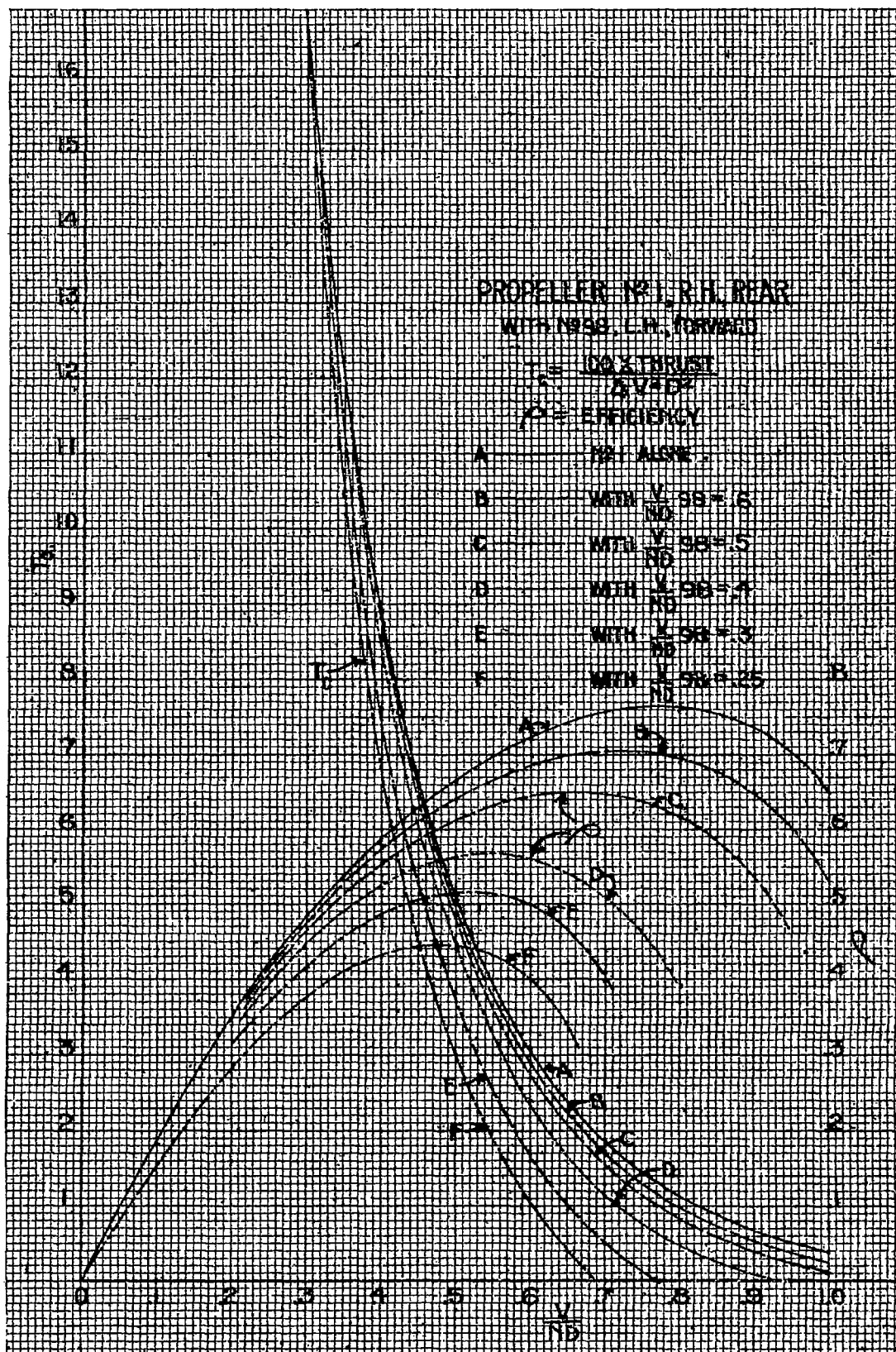


PLATE XVII.

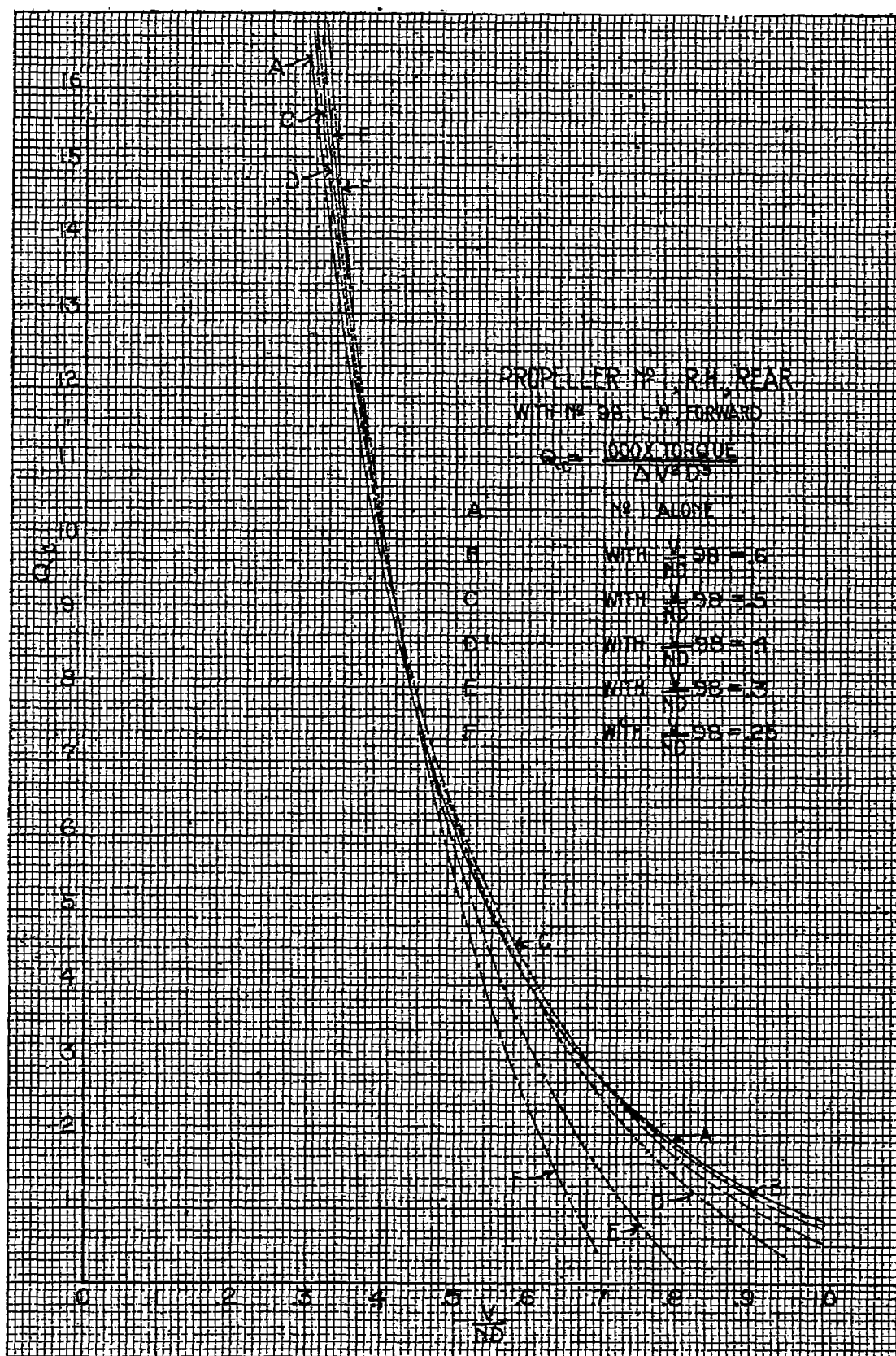


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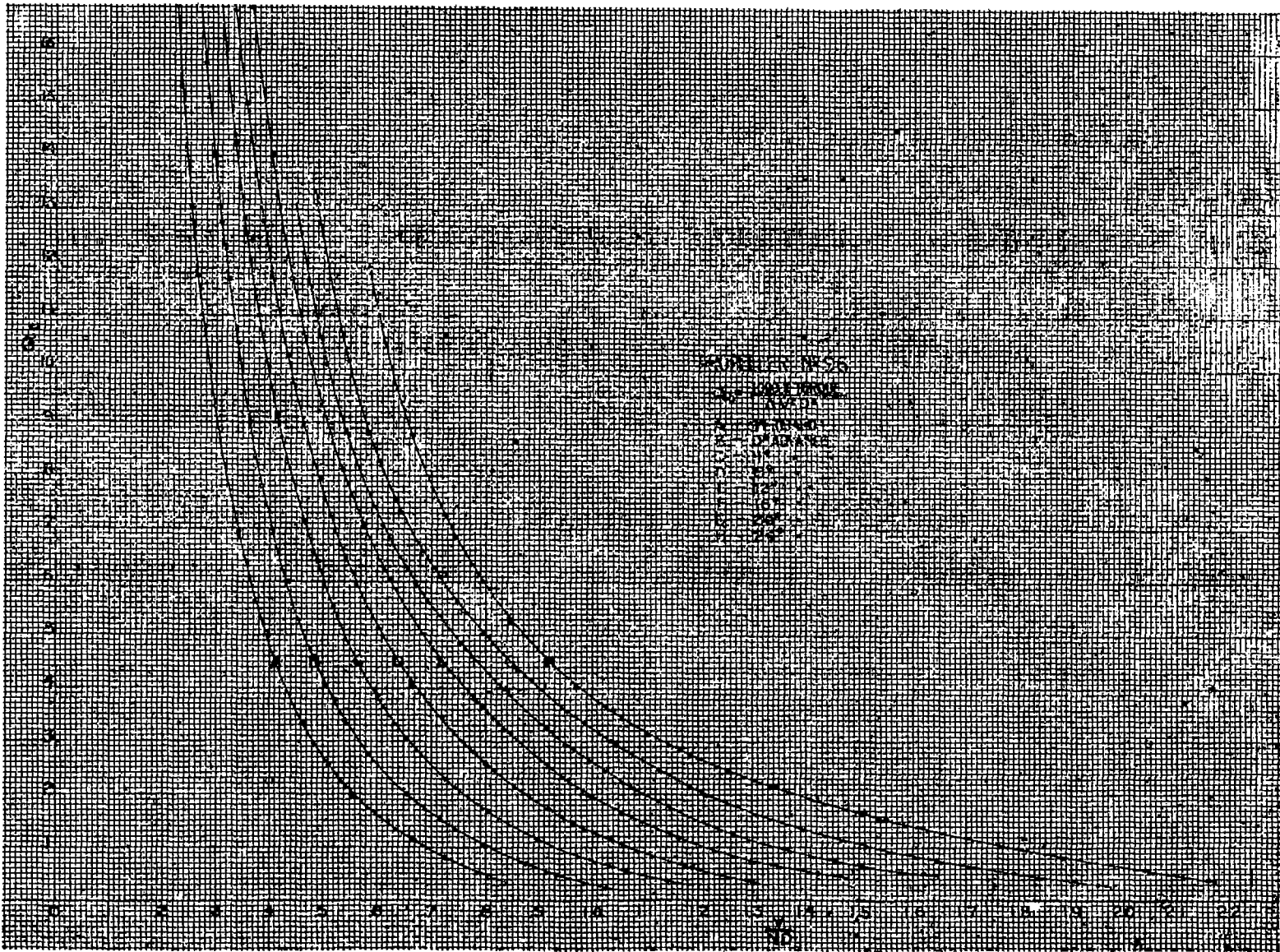


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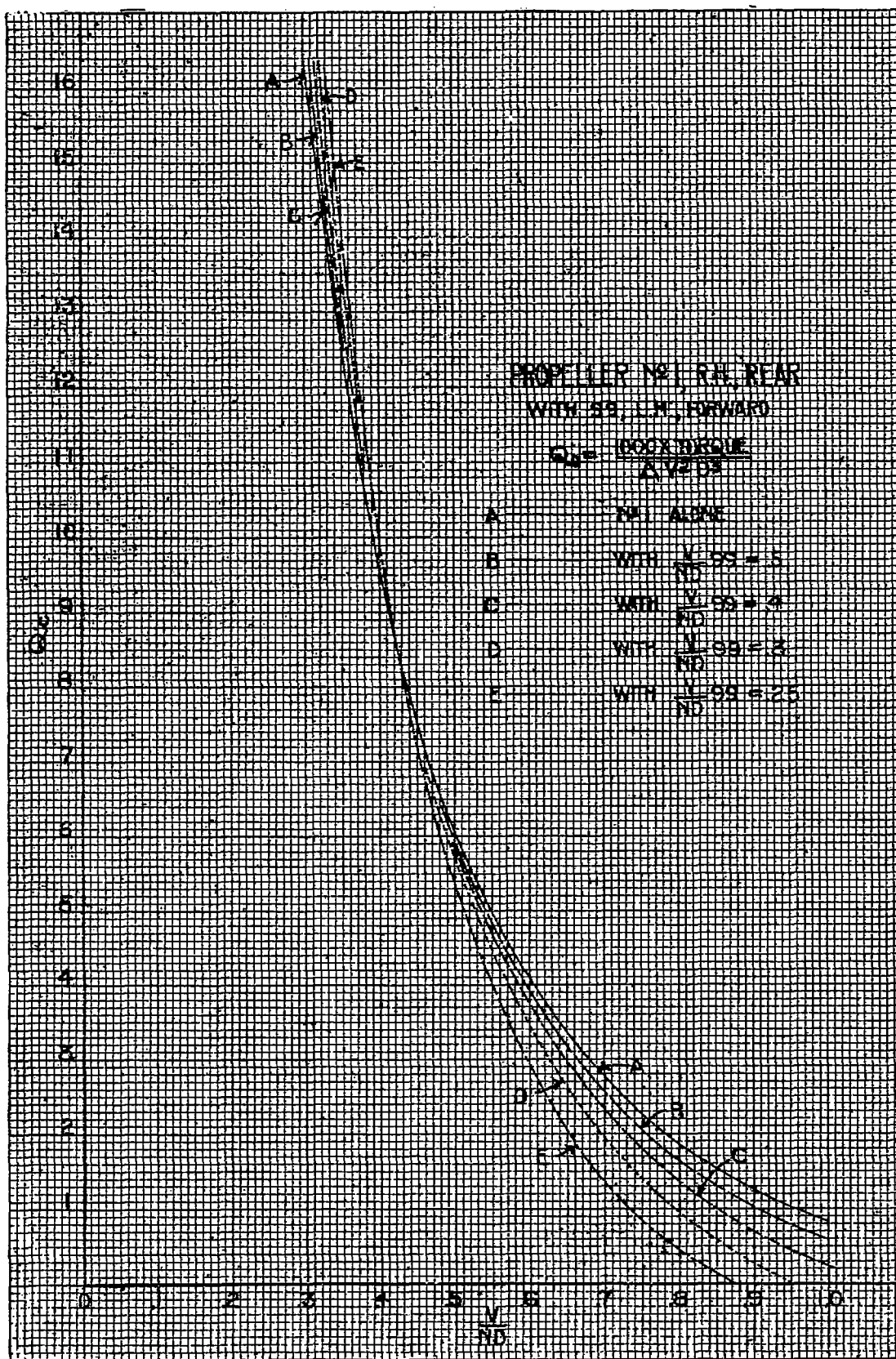


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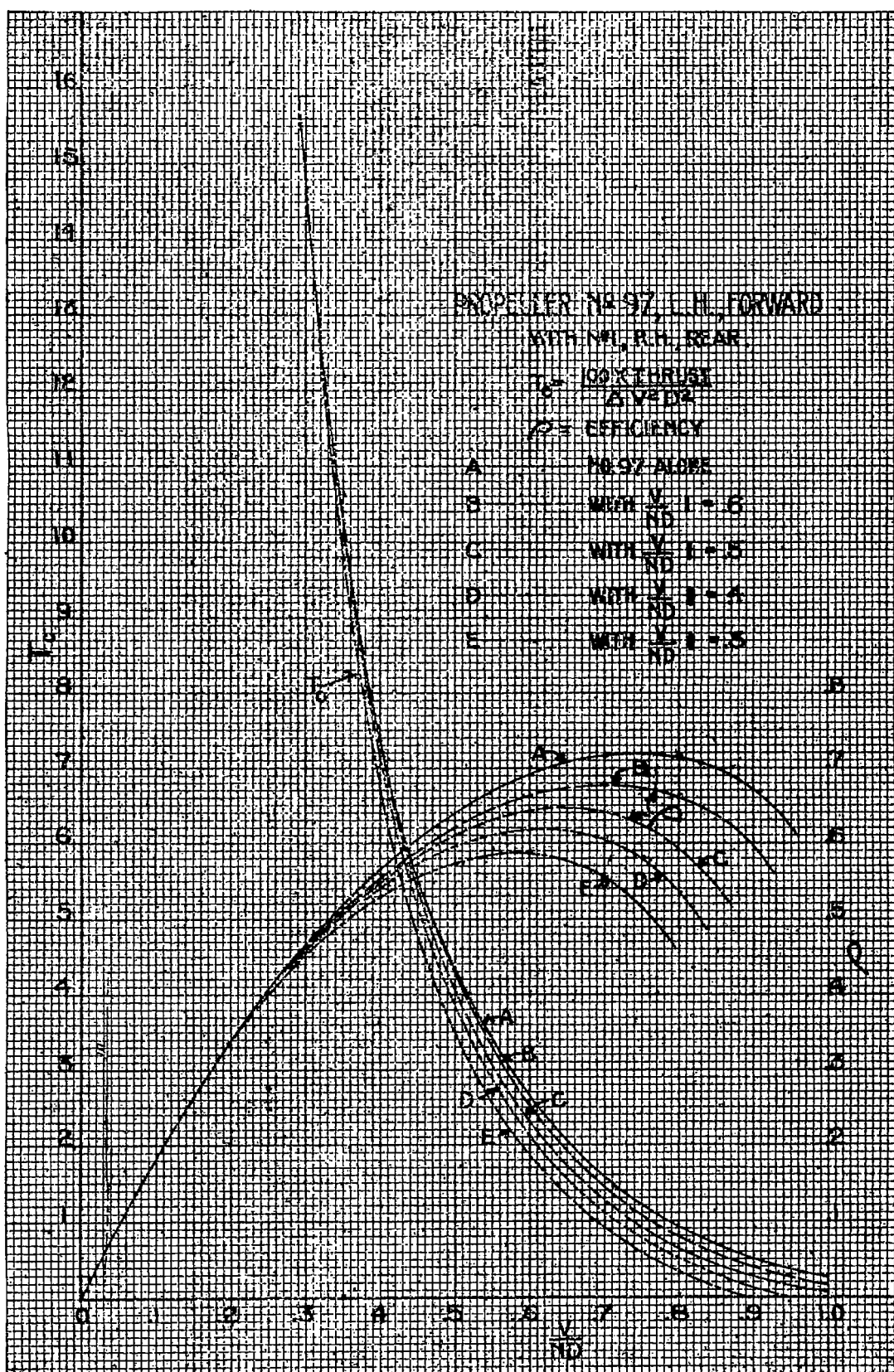


PLATE XXI.

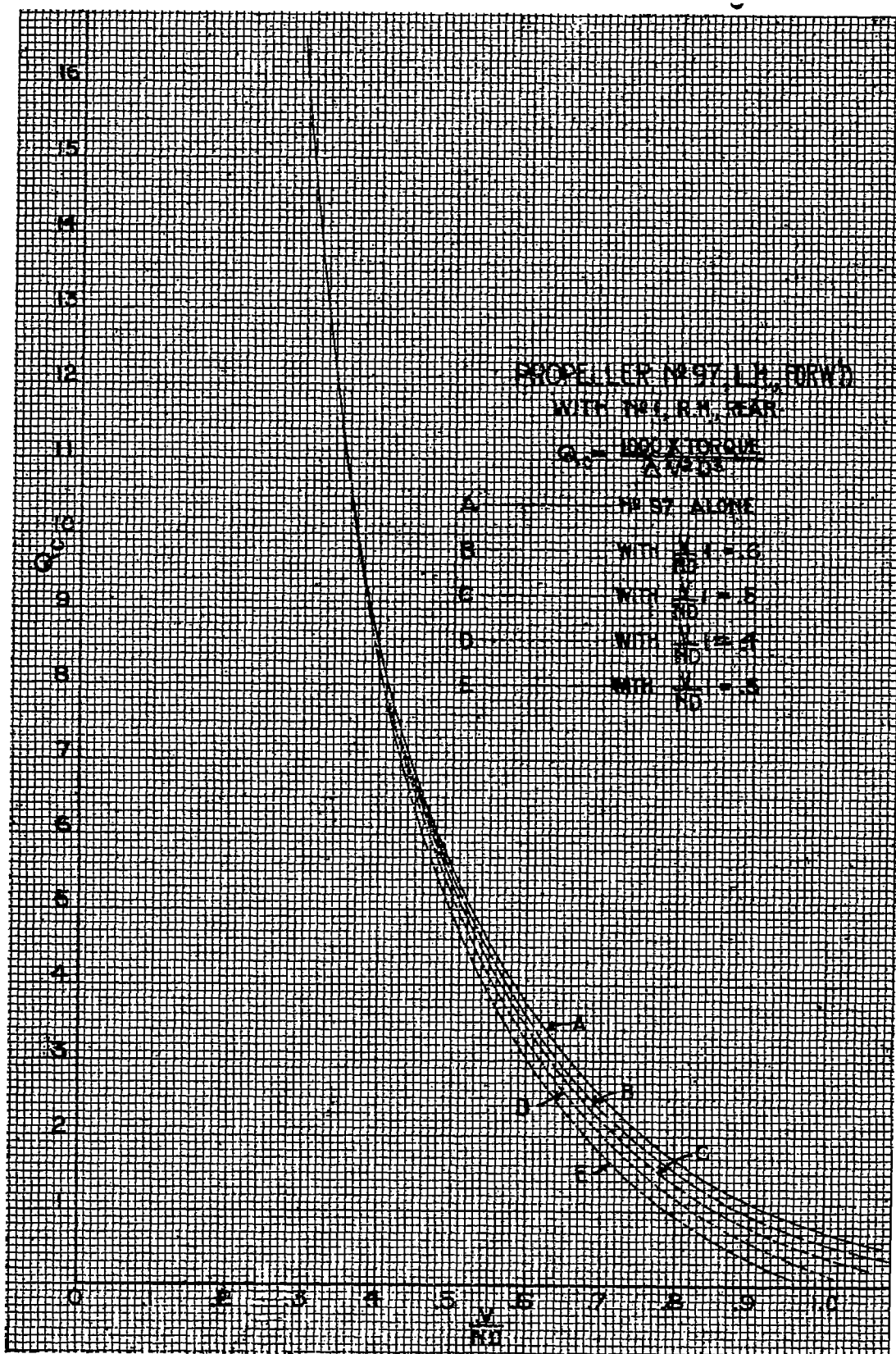


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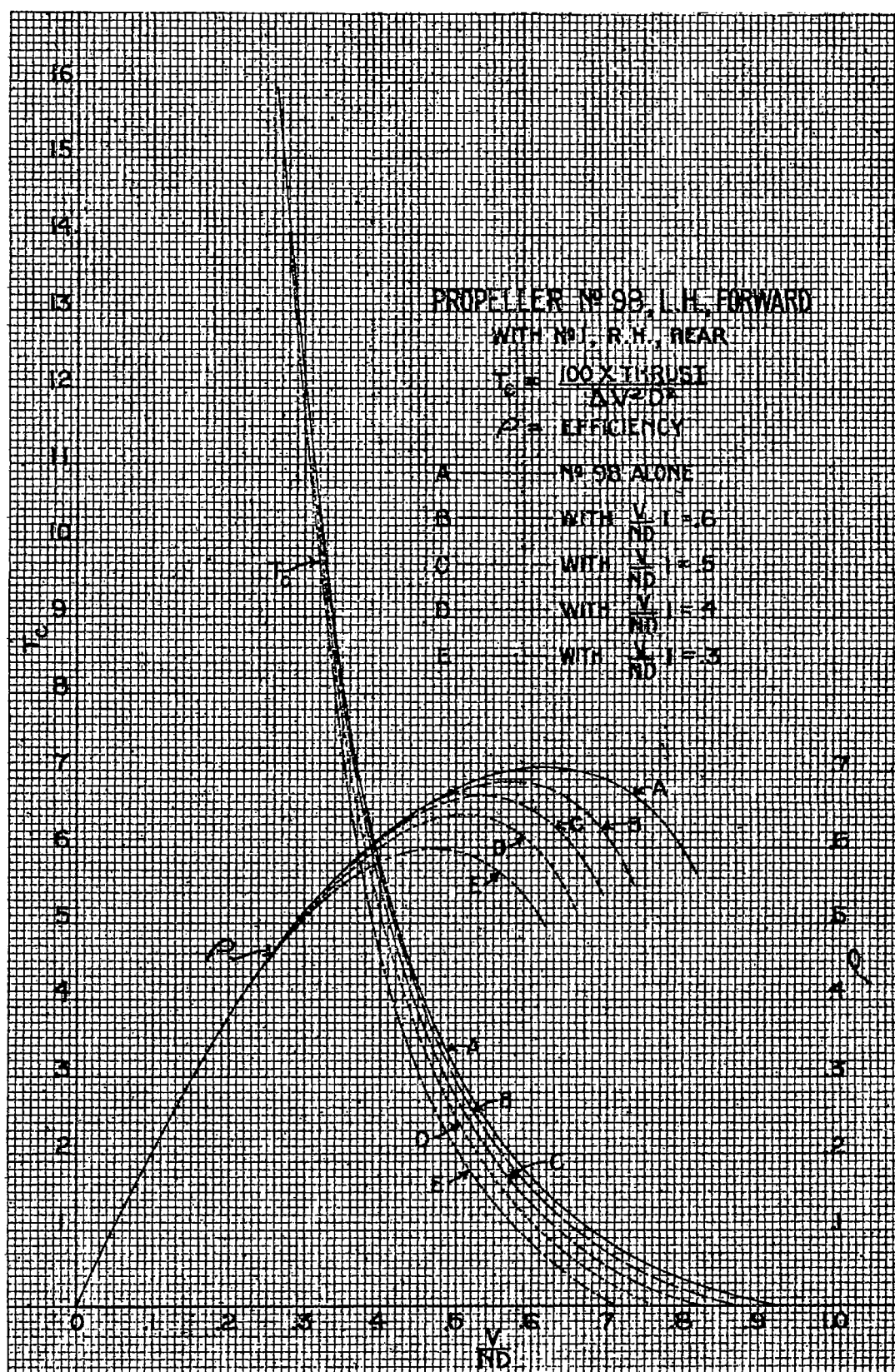


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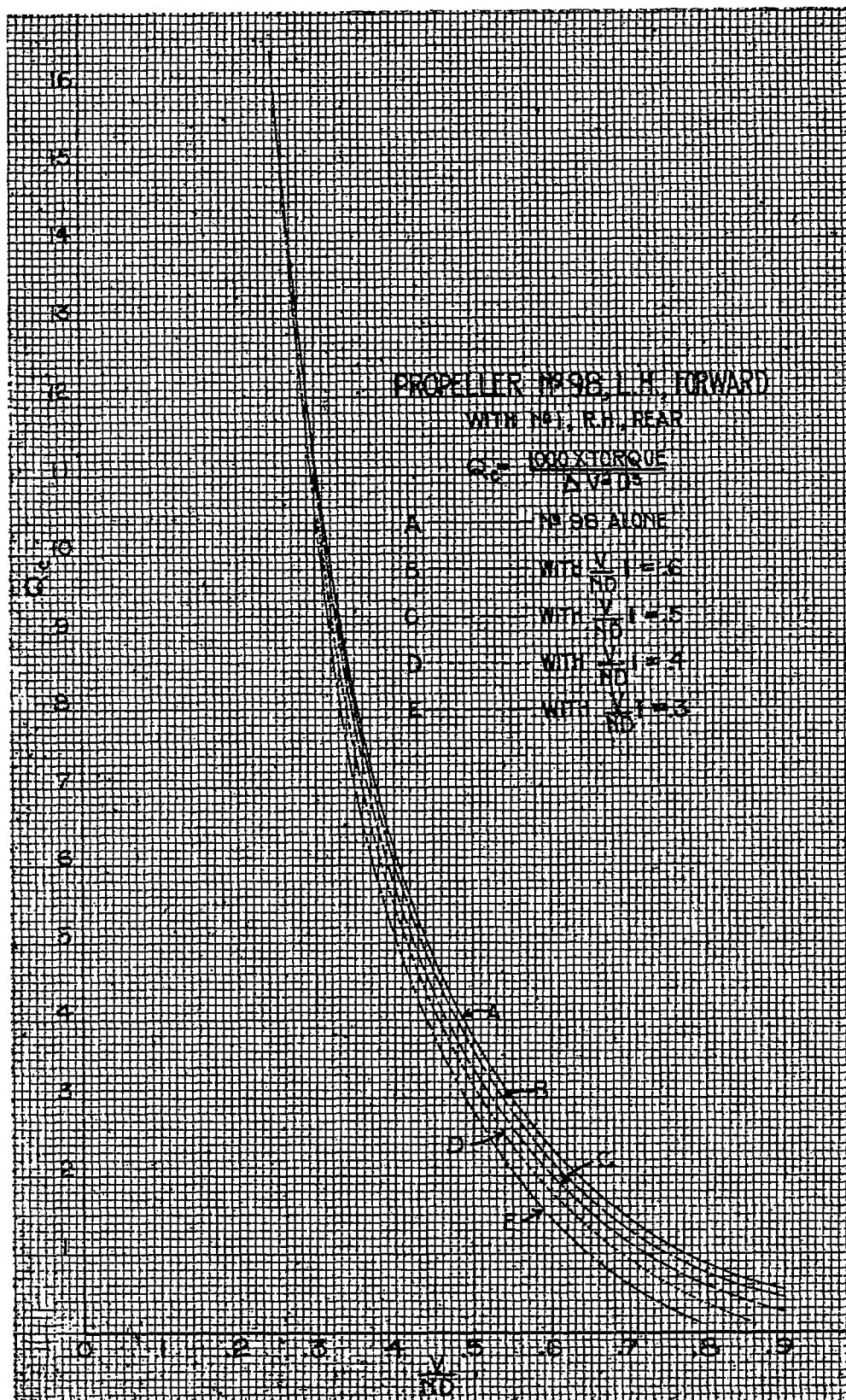


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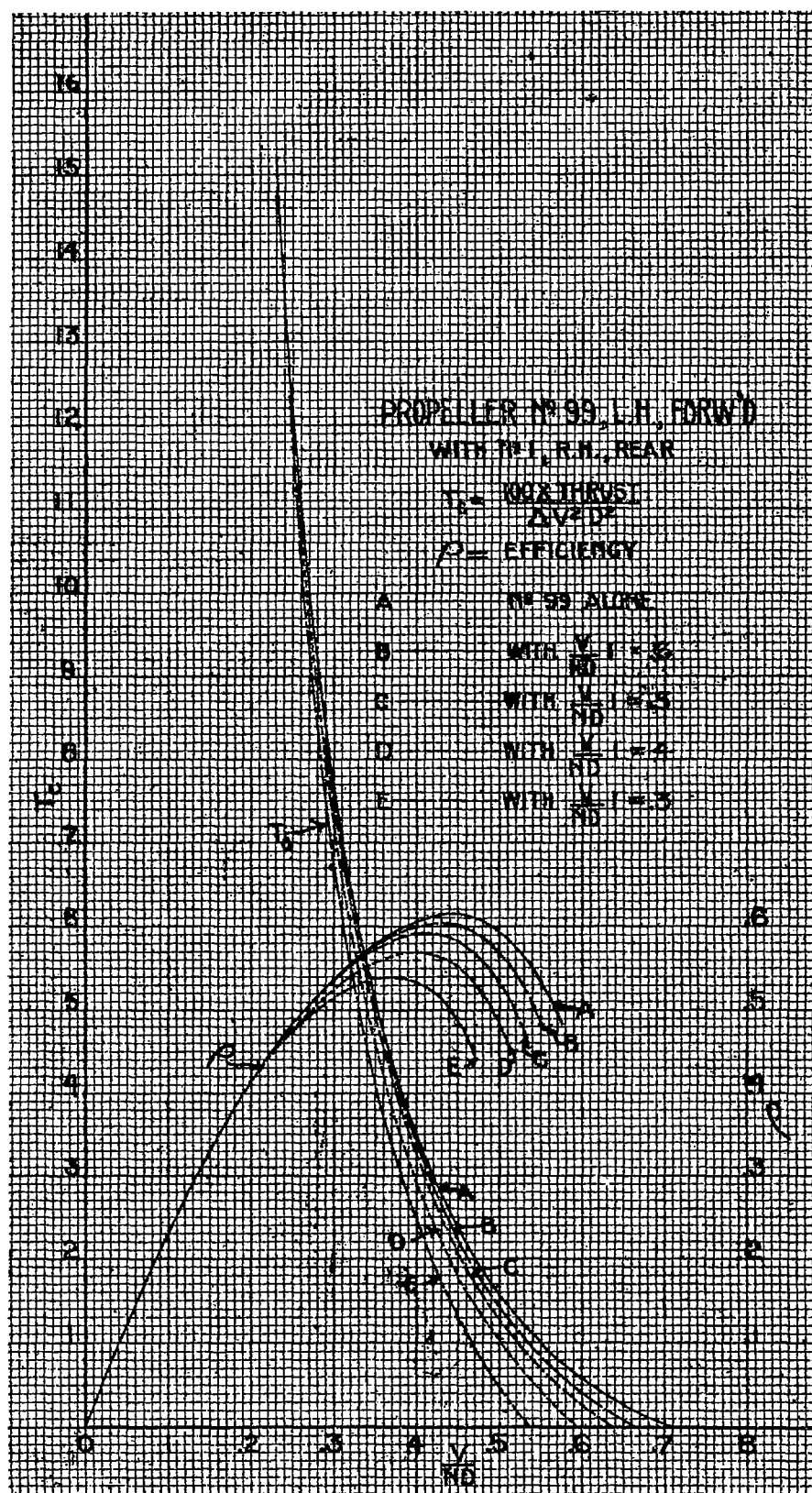


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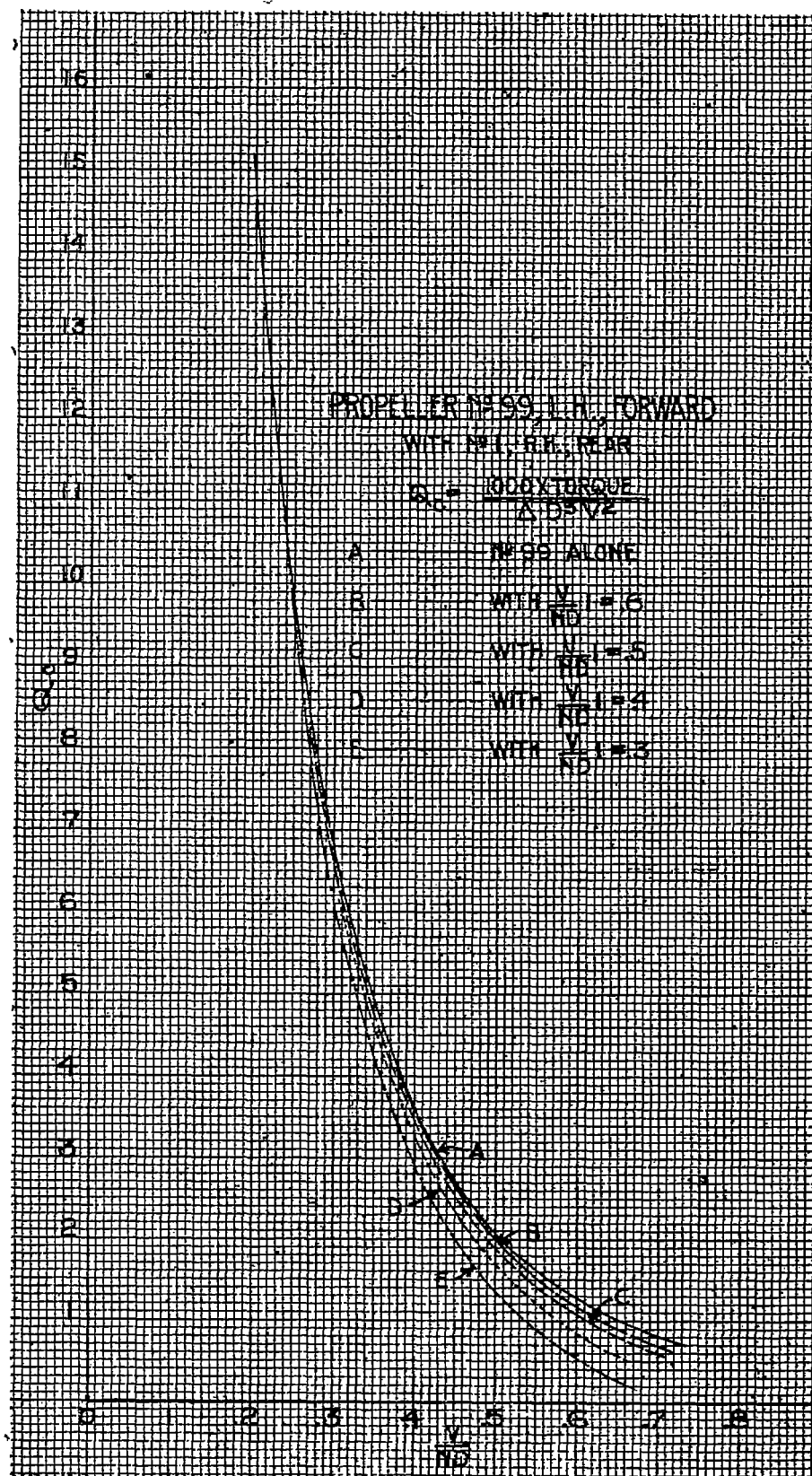


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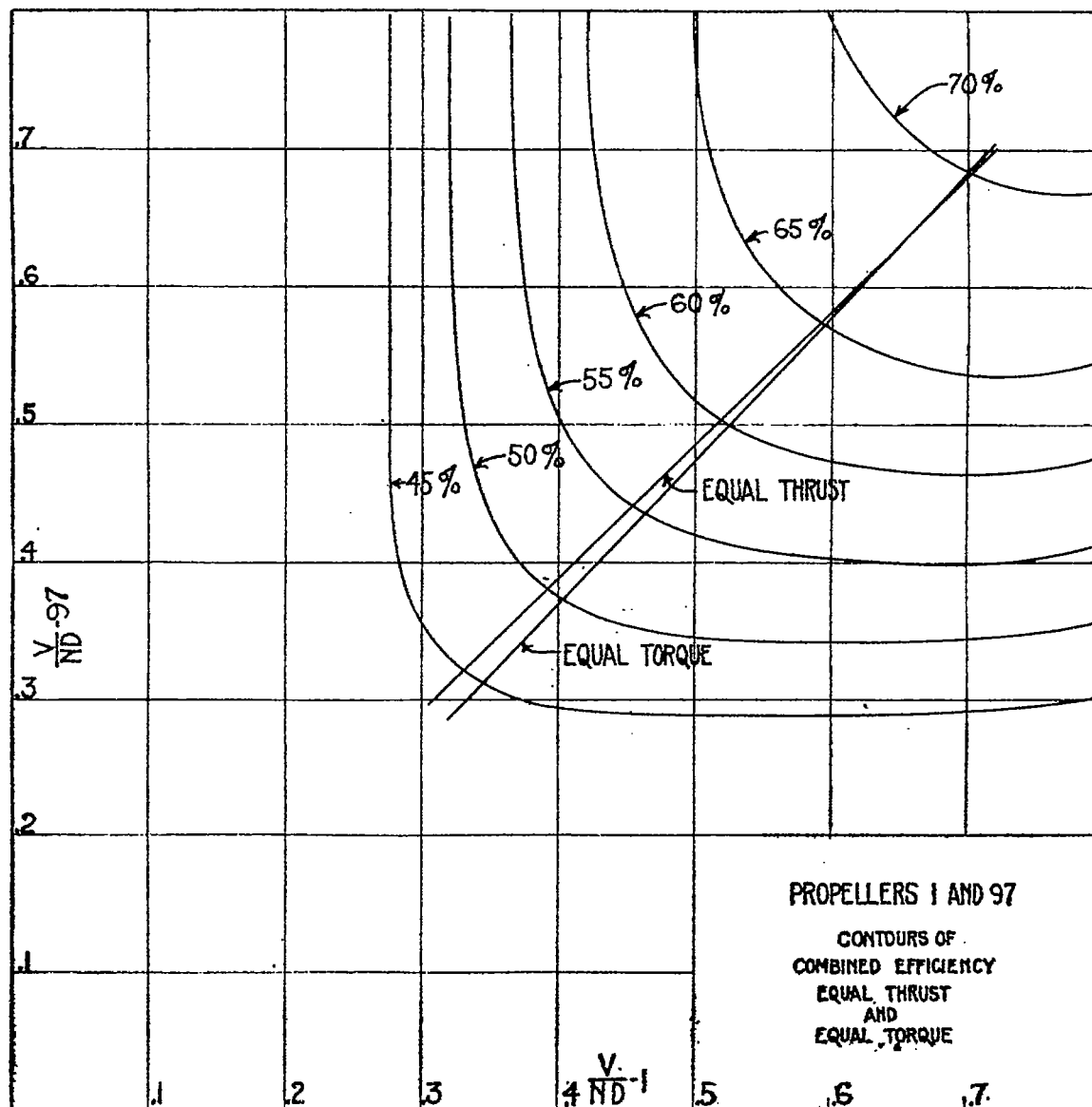


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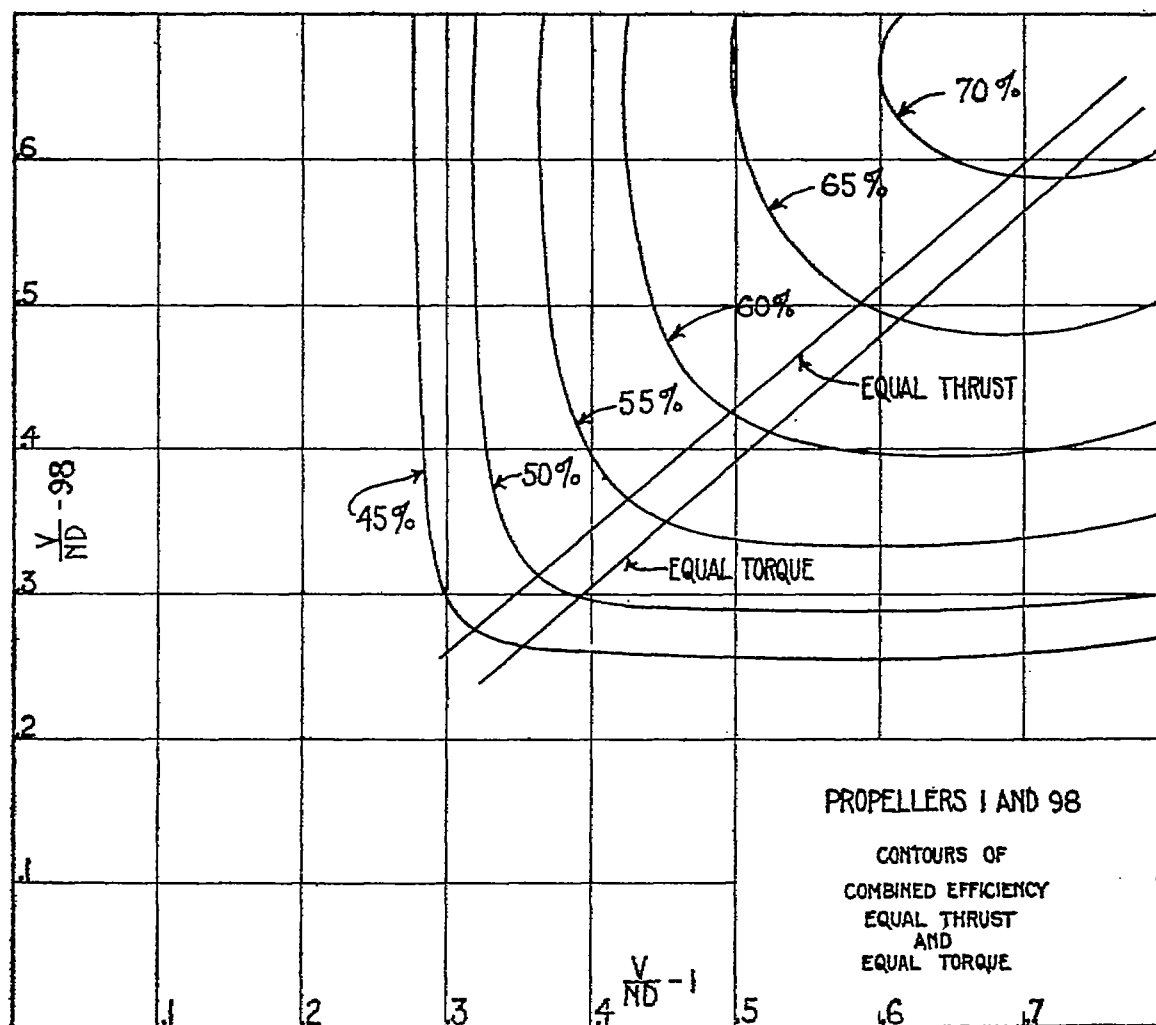


PLATE XXVIII.

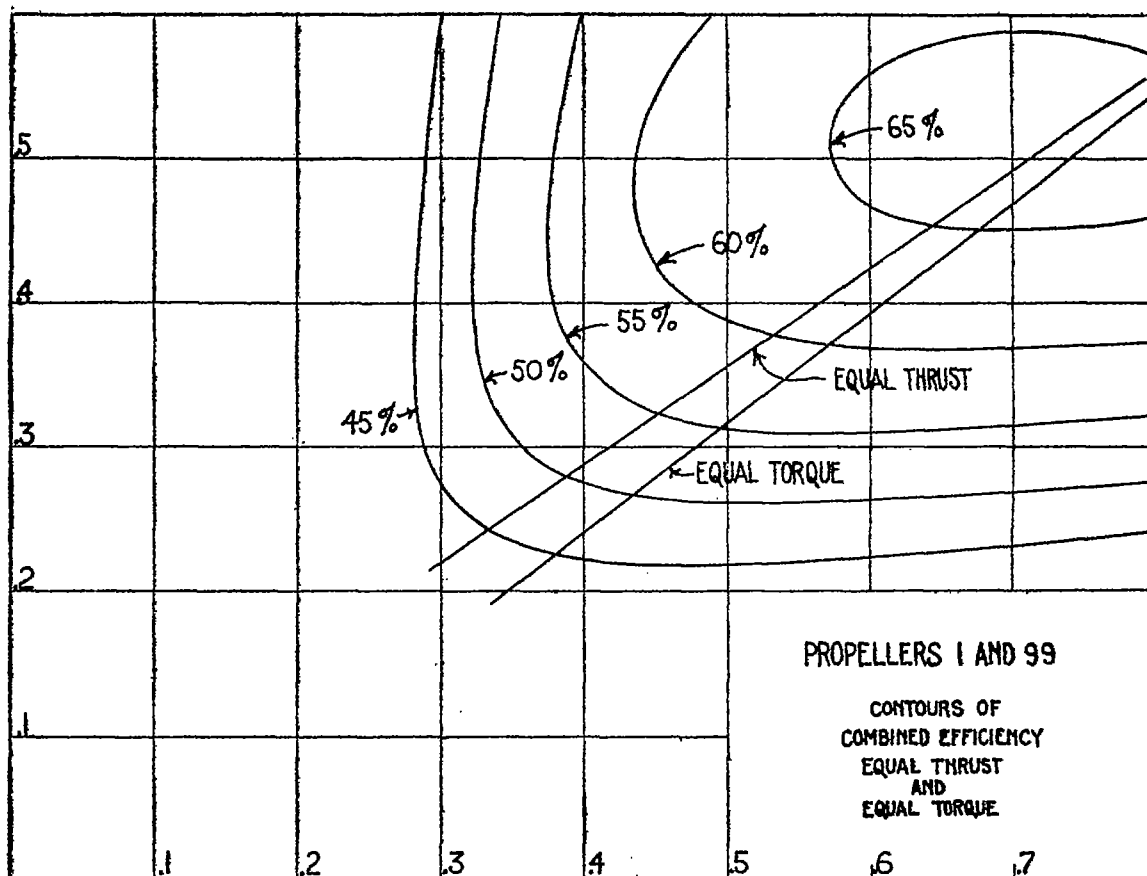


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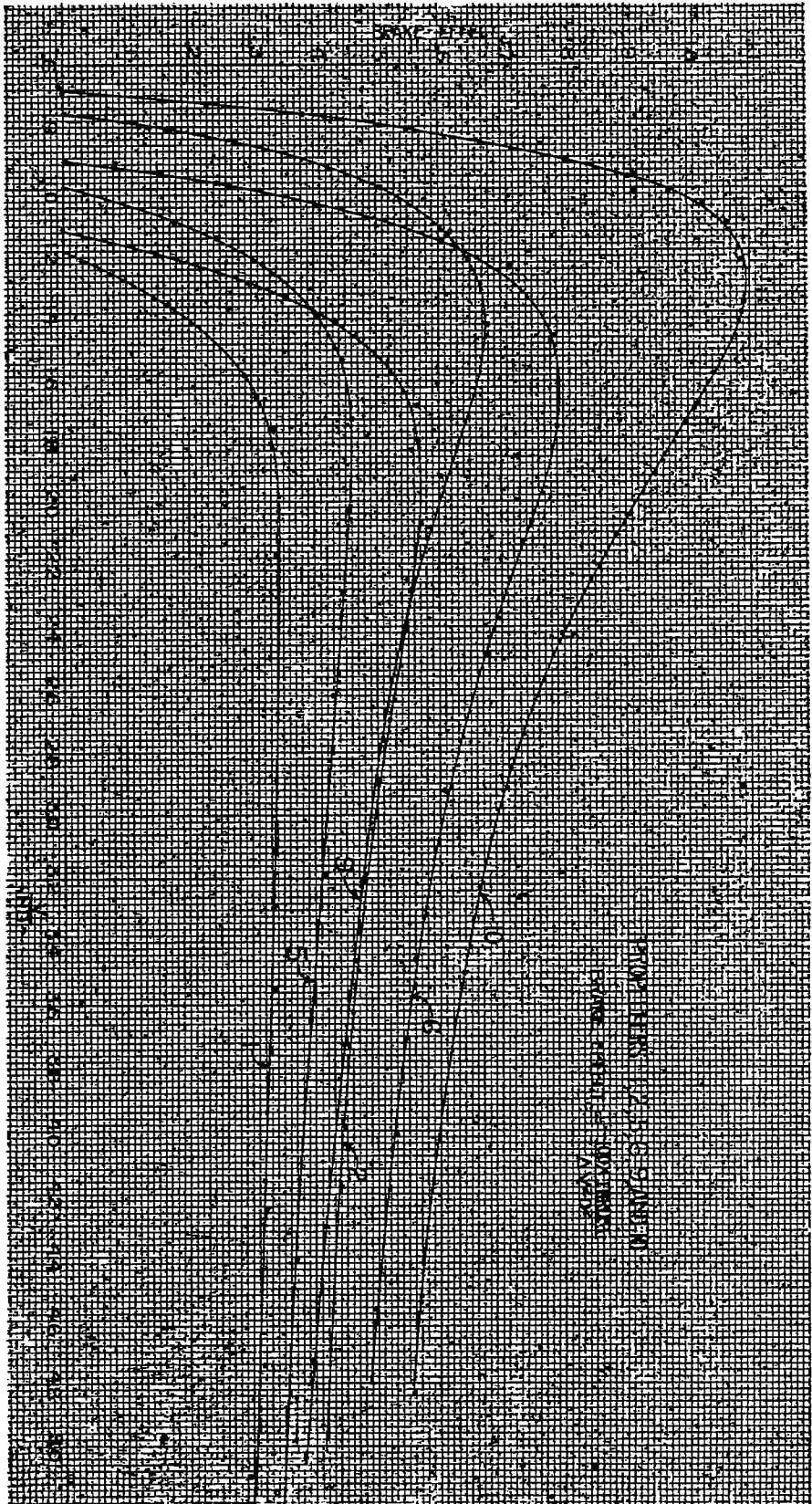


PLATE XXX.

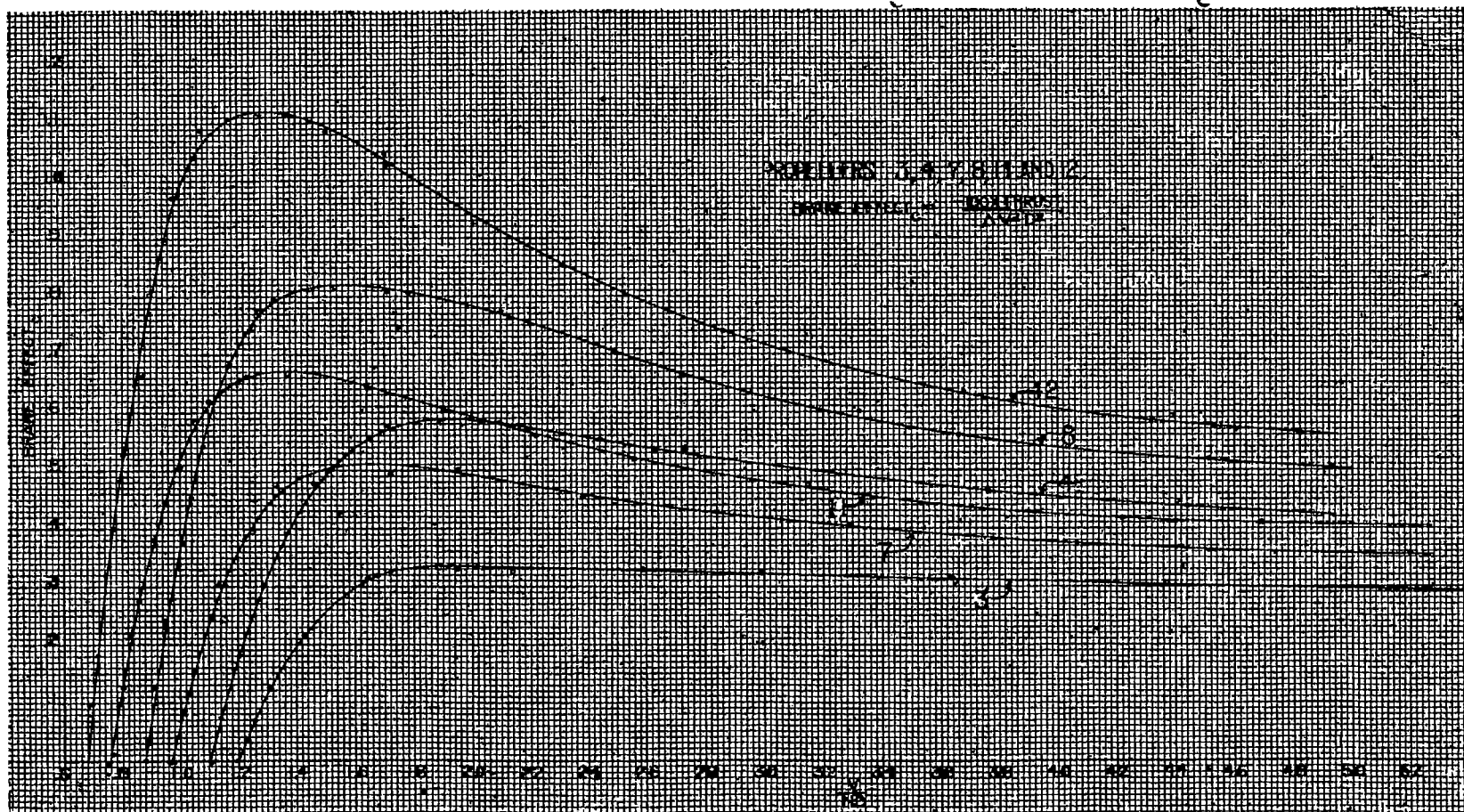


PLATE XXXI.

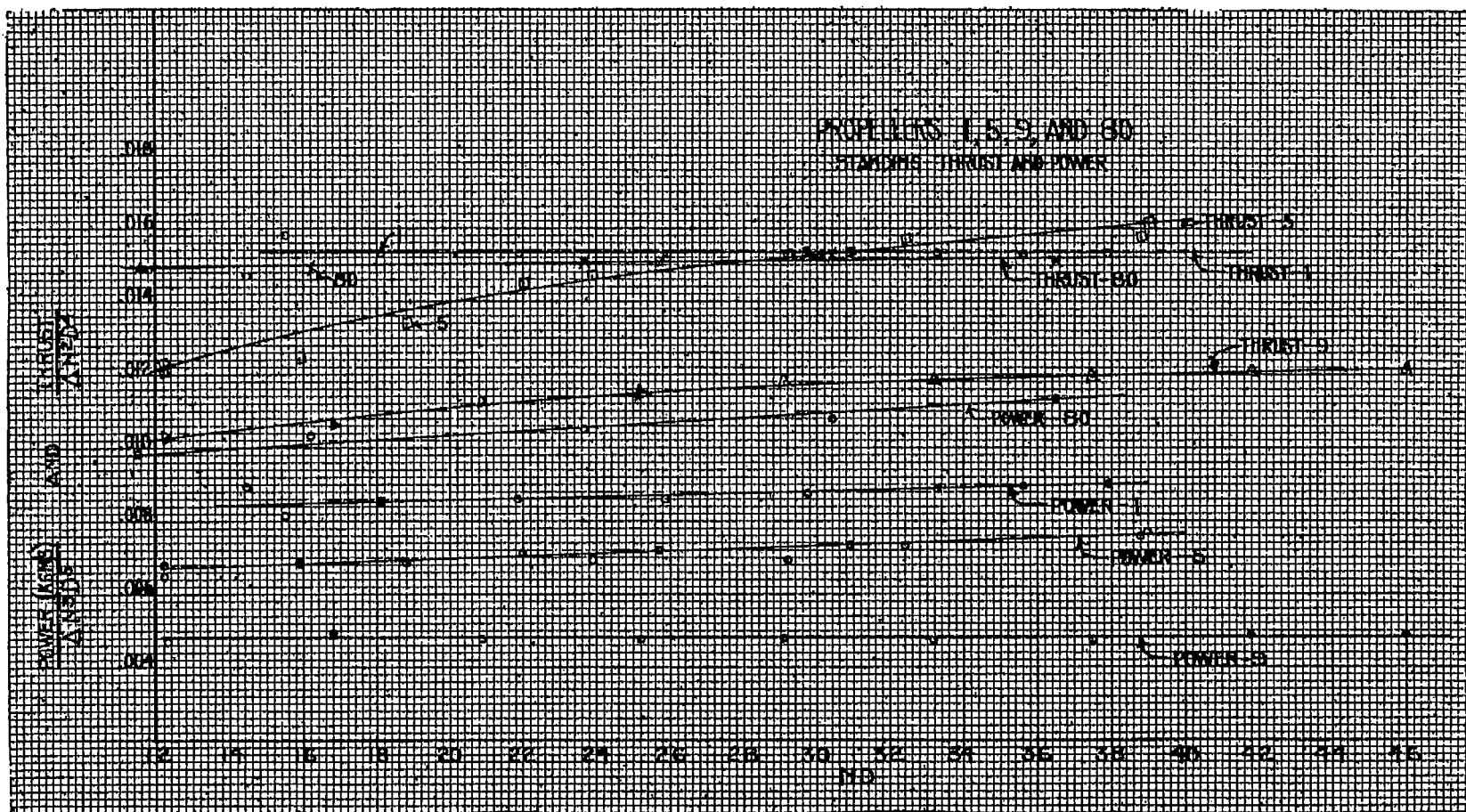


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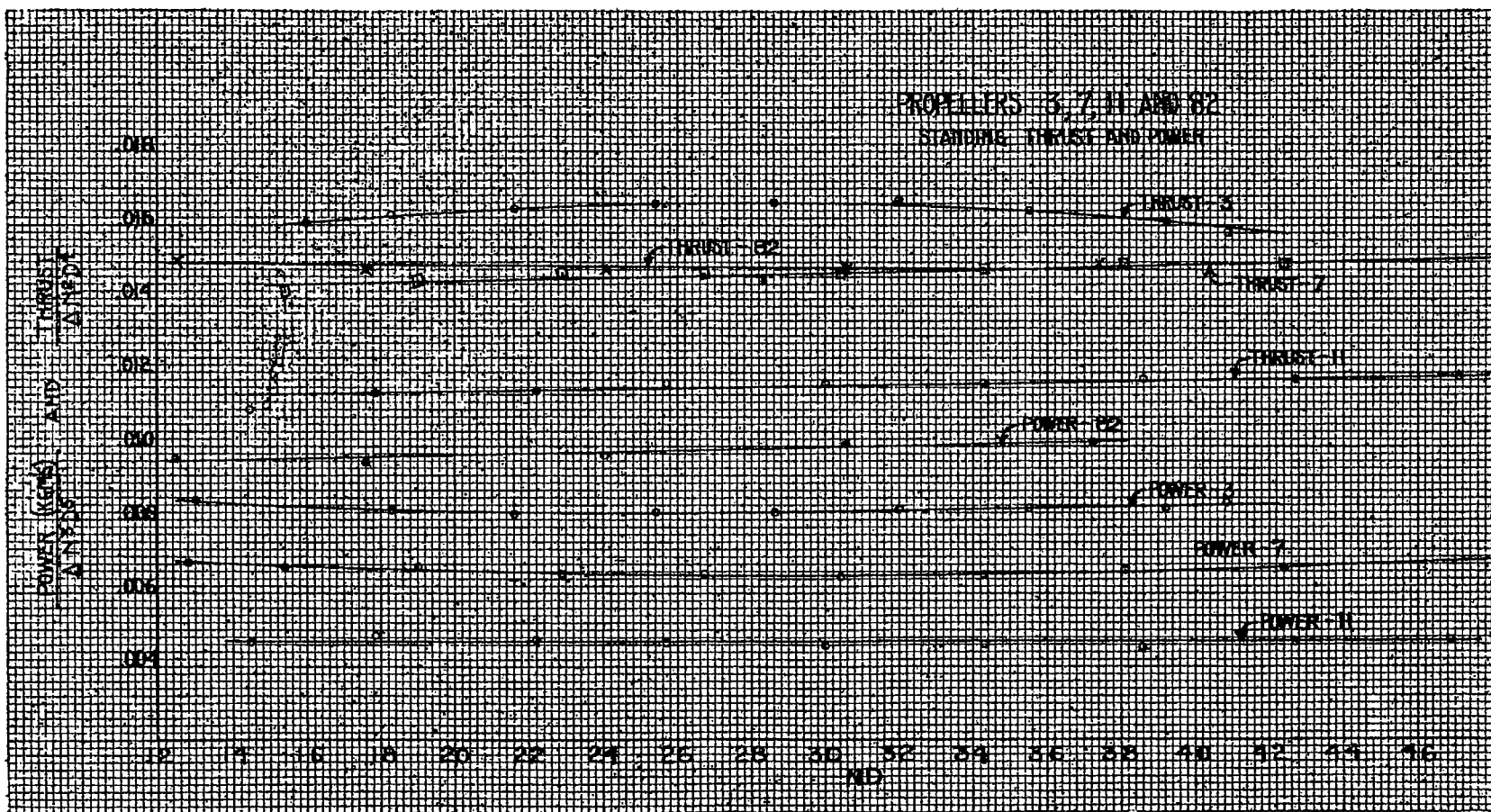


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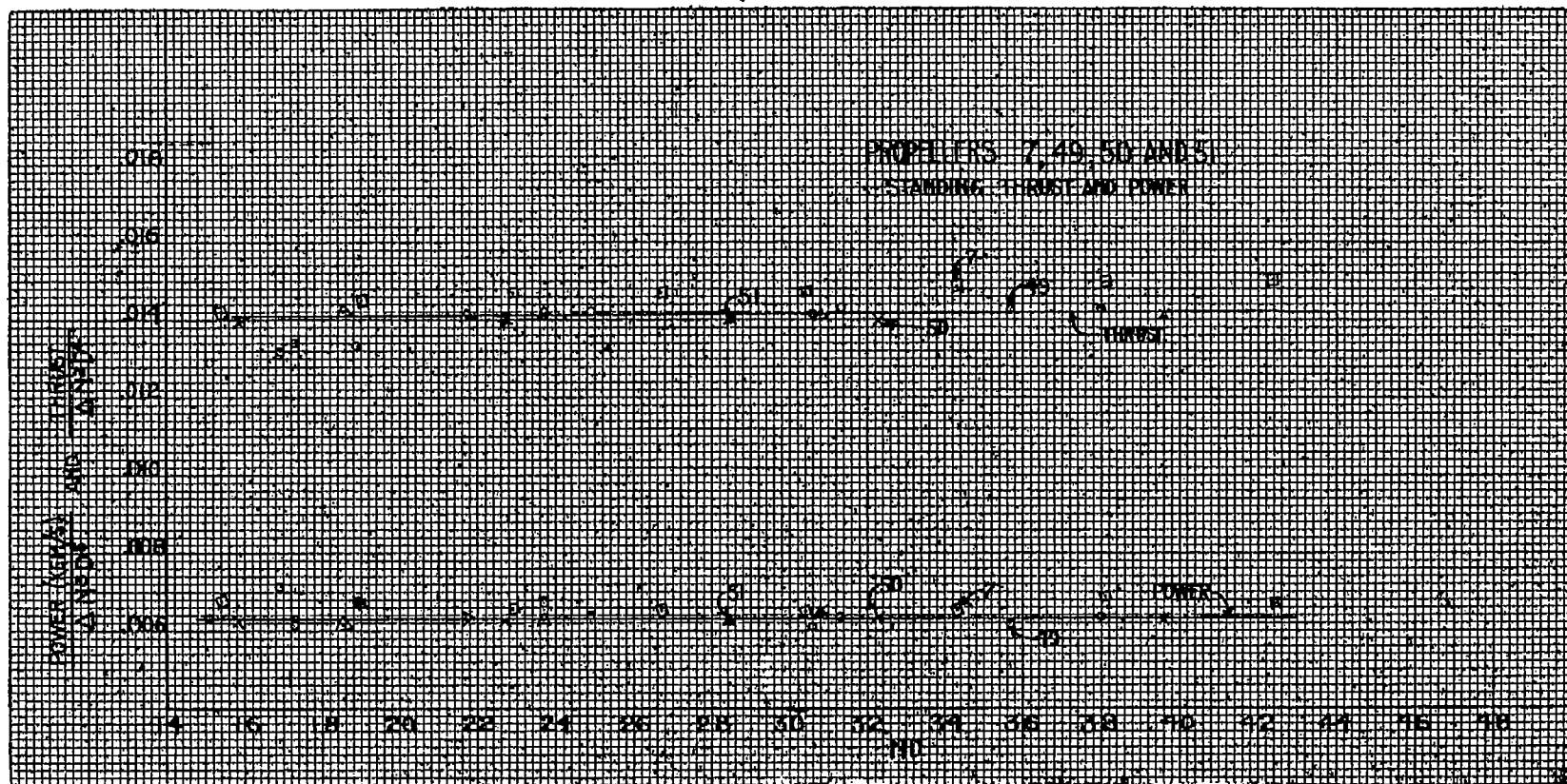
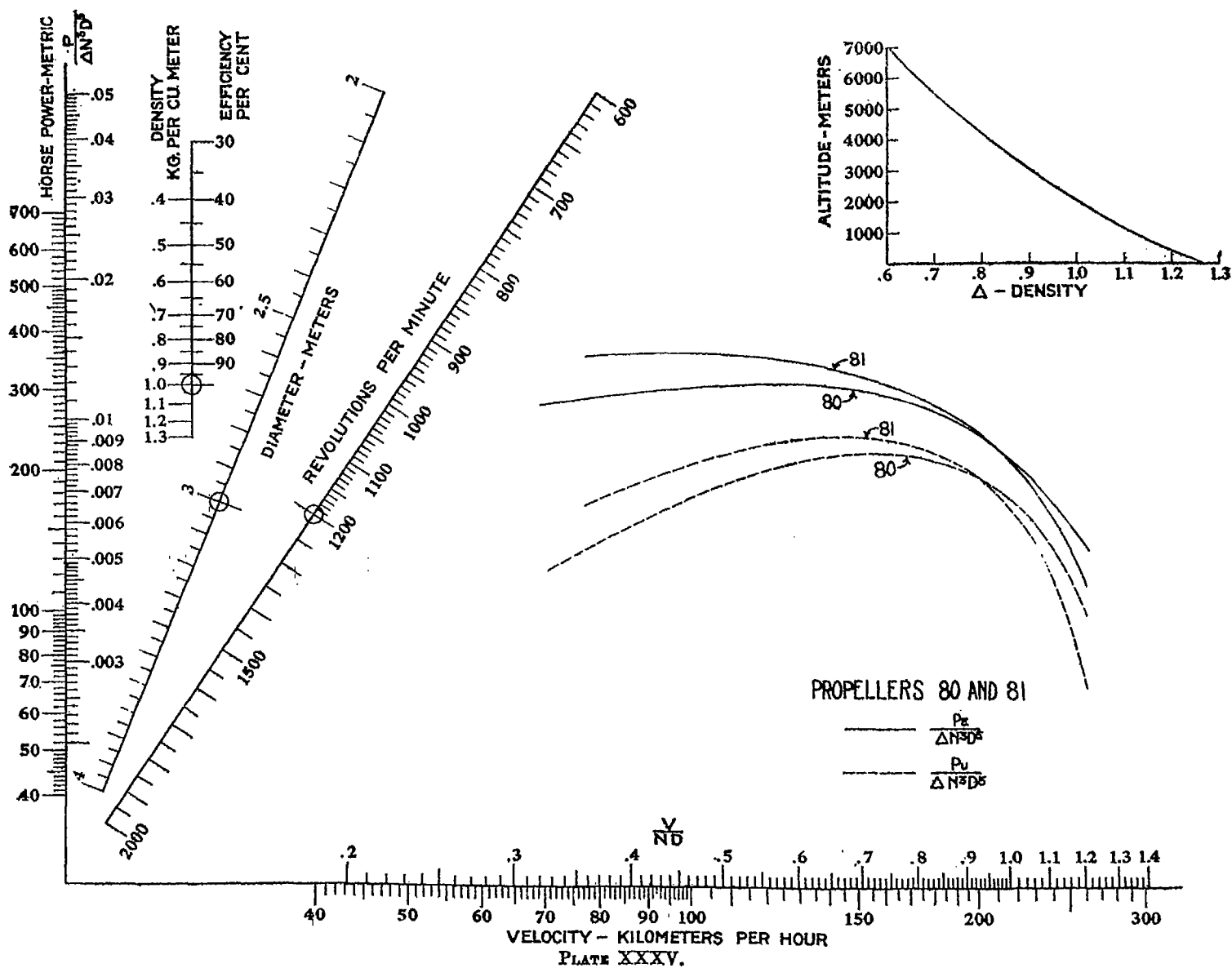
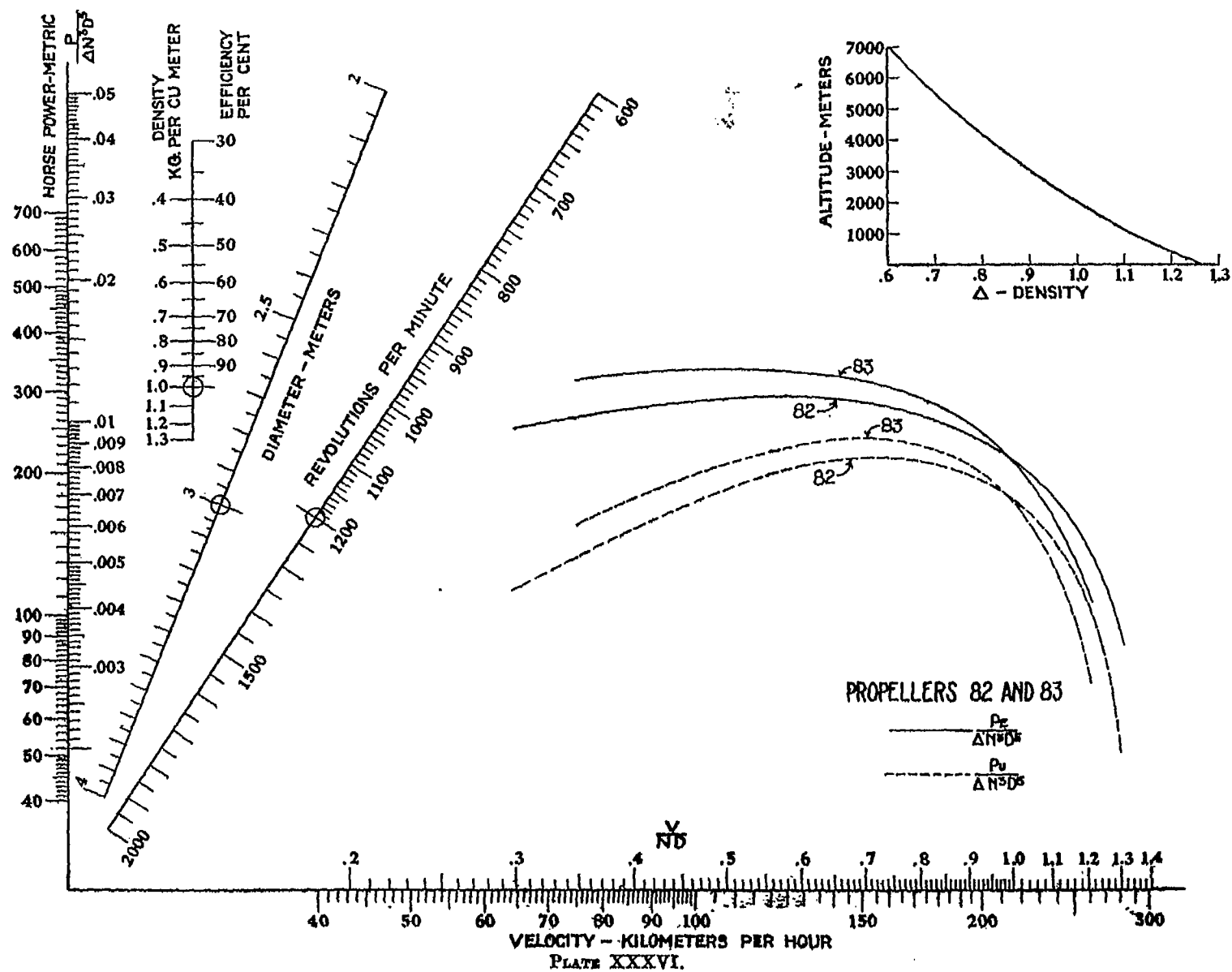
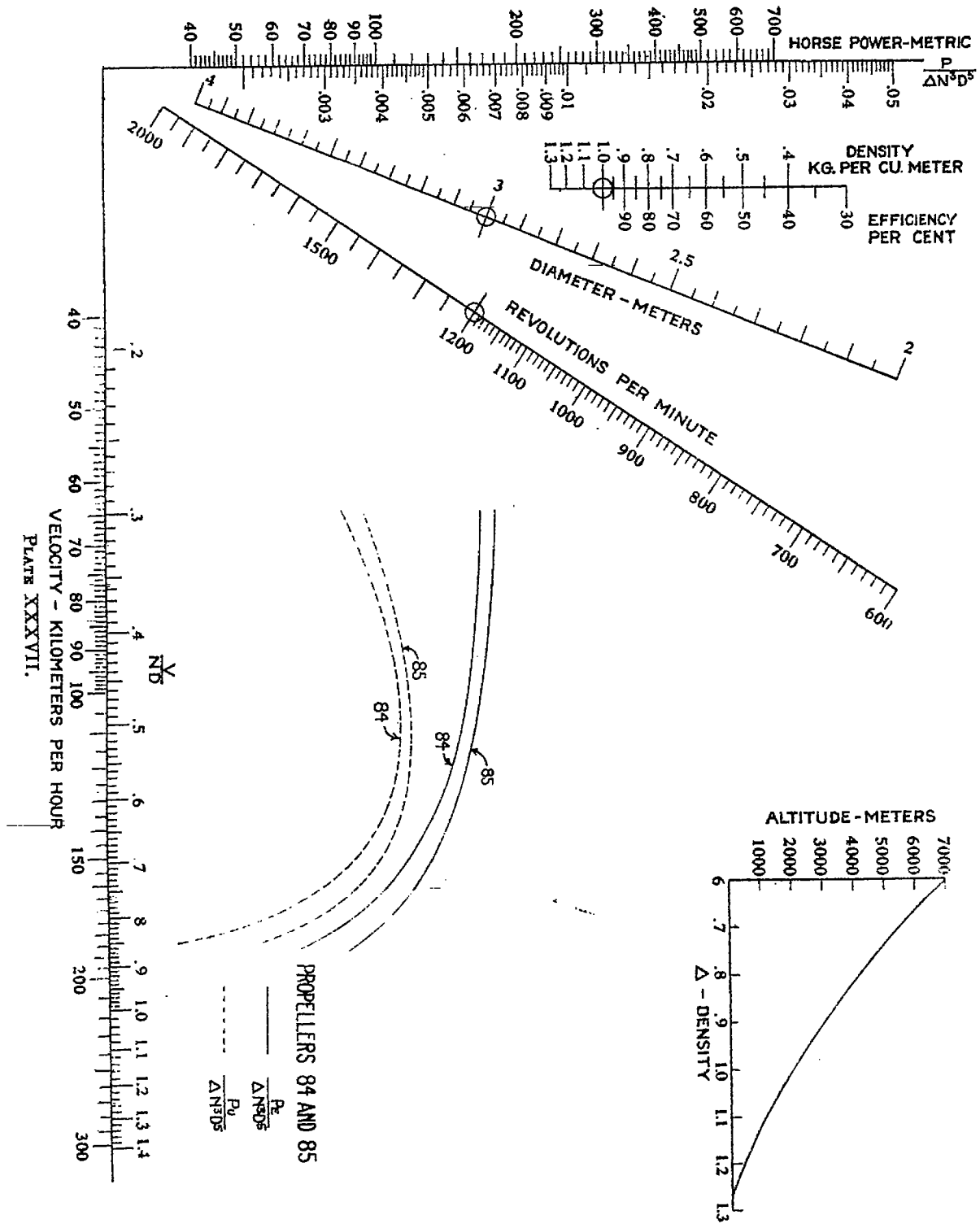


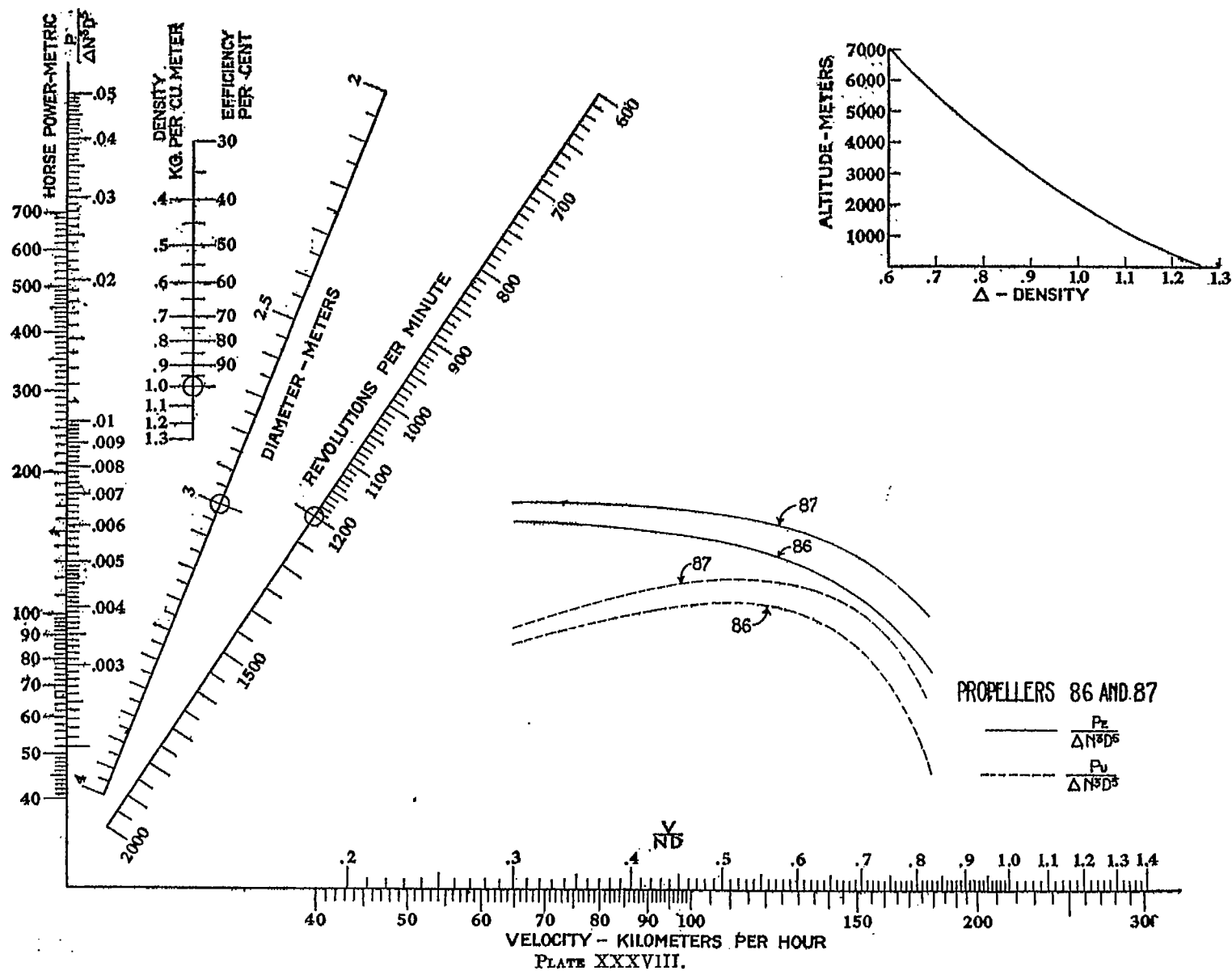
PLATE XXXIV.











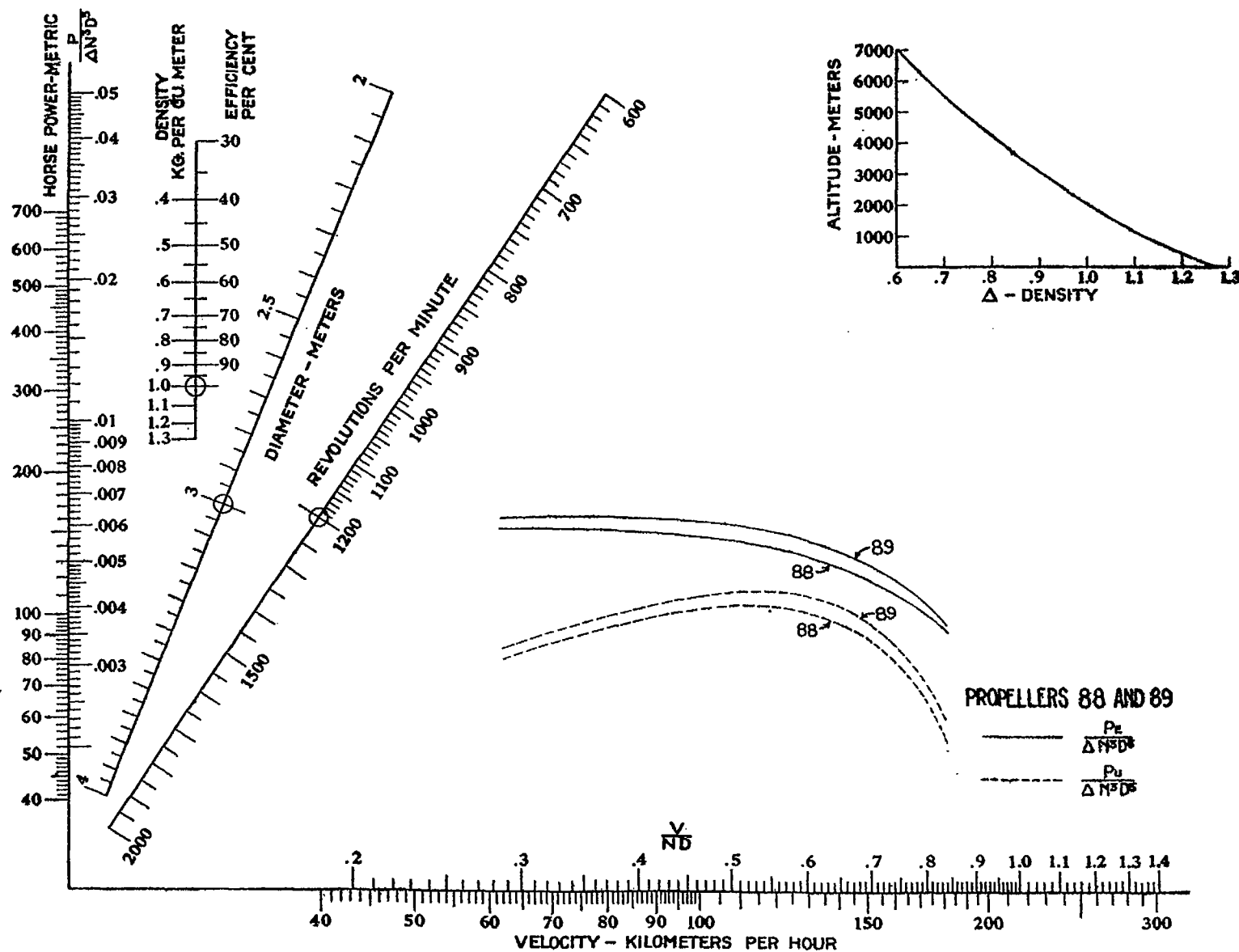


PLATE XXXIX.

